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ABSTRACT

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The dispersion study of a TID event has been used to illustrate the possibility of deriving neutral atmospheric structure from such type studies.

The use of a parametric model to describe the power spectra of the phase-path fluctuations was investigated. The results indicate that the background activity in the 1 CPH to 13 CPH frequency range follows an excitation mechanism having 12- and 24-hour components.

Phase-path records corresponding to high energy events were studied. It was found that identification of signals coming from the events is very difficult due to the large increase of background activity at the expected signal arrival time.



FINAL REPORT
PROPAGATION OF LONG PERIOD ACOUSTIC
GRAVITY WAVES IN THE ATMOSPHERE

HERBANO R. MENDES

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January 31, 1972

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ABSTRACT

Atmospheric wave data from a multisensor array have been analyzed. It was found that Saturn Apollo launches generate acoustic gravity waves which propagate in the upper atmosphere and induce phase-path fluctuations.

The dispersion study of a TID event has been used to illustrate the possibility of deriving neutral atmospheric structure from such type of studies.

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Phase-path records corresponding to high energy events were studied. It was found that identification of signals coming from the events is very difficult due to the large increase of the background activity at the expected signal arrival time.

1. INTRODUCTION

The objective of this project was to analyze atmospheric wave data collected with a multisensor array during 1969-1970. The multisensor array consisted of the superposition of long period microbarographs, magnetometers and phase-path (doppler) sounders, and was situated in the New York-New Jersey region. The analyses have been directed, in particular, to those events, natural or man-made, from which atmospheric structure information could be obtained.

Data recorded during Saturn-Apollo launchings have been extensively analyzed to see if any evidence of coupling between the upper and lower part of the atmosphere could be found. A study of other rocket launchings was carried out to determine if these rockets also produced doppler signatures similar to those observed during Apollo launchings.

Some Traveling Ionospheric Disturbances (TIDs) were observed during the operation of the phase-path sounder array. The dispersion study of the TID event of 23 November 1969 has been completed and is illustrated here.

A statistical model of the ionospheric background motions has been constructed. This work was done in cooperation with Dr. Melvin Hinich of the Department of Statistics, Carnegie-Mellon University, Pittsburgh. The model describes the power spectrum of the phase-path fluctuations as a function of frequency and time. The parameters of the model have been determined by multi-regression analyses of hundreds of power spectra. The effect of sampling on the model has also been studied.

A simplified theoretical study of the ionospheric response to internal gravity waves was carried out. The approach to this study has been from the point of view of the doppler observations.

Finally, the records corresponding to the high energy events of the summer of 1970 have been studied to look for possible signatures.

2. ROCKET-GENERATED SIGNATURES ON PHASE-PATH RECORDS

Introduction

Phase-path (doppler) variations induced by very low frequency (0.004167 Hz - 0.0167 Hz) acoustic-gravity waves generated by large rockets were observed with an array of phase-path sounders at the times of the Saturn-Apollo 12 and 13 launches. Array processing of these signals showed that the average horizontal phase velocities for these events were of the order of 720 m sec^{-1} and the arrival azimuth about 175° (Tolstoy et al., 1970; Montes and Posmentier, 1971).

Since one or two phase-sounders and some microbarographs were also in operation during the launches of Saturn-Apollo 8, 9, 10, and 11, it was decided to review the phase-path, magnetometer and microbarograph records to investigate if all Apollo launches have generated signals of this kind and if the signals were present in the microbarograph and magnetometer records. In addition, we have also searched the records at times corresponding to other rocket launchings.

Data Analyses and Results

The data used in this investigation were obtained in the New York-New Jersey area with a low-frequency multisensor array consisting of microbarographs, magnetometers and phase-path sounders (Fig. 1). The instrumentation has been described in a previous report (Montes et al., 1970). The overall system operated with an approximately flat response in the frequency range 0.05 Hz to 0.0002778 Hz.

The phase-path data were recorded using phase-path (doppler) sounders essentially of the type described by Davies (1962) but with slight modifications

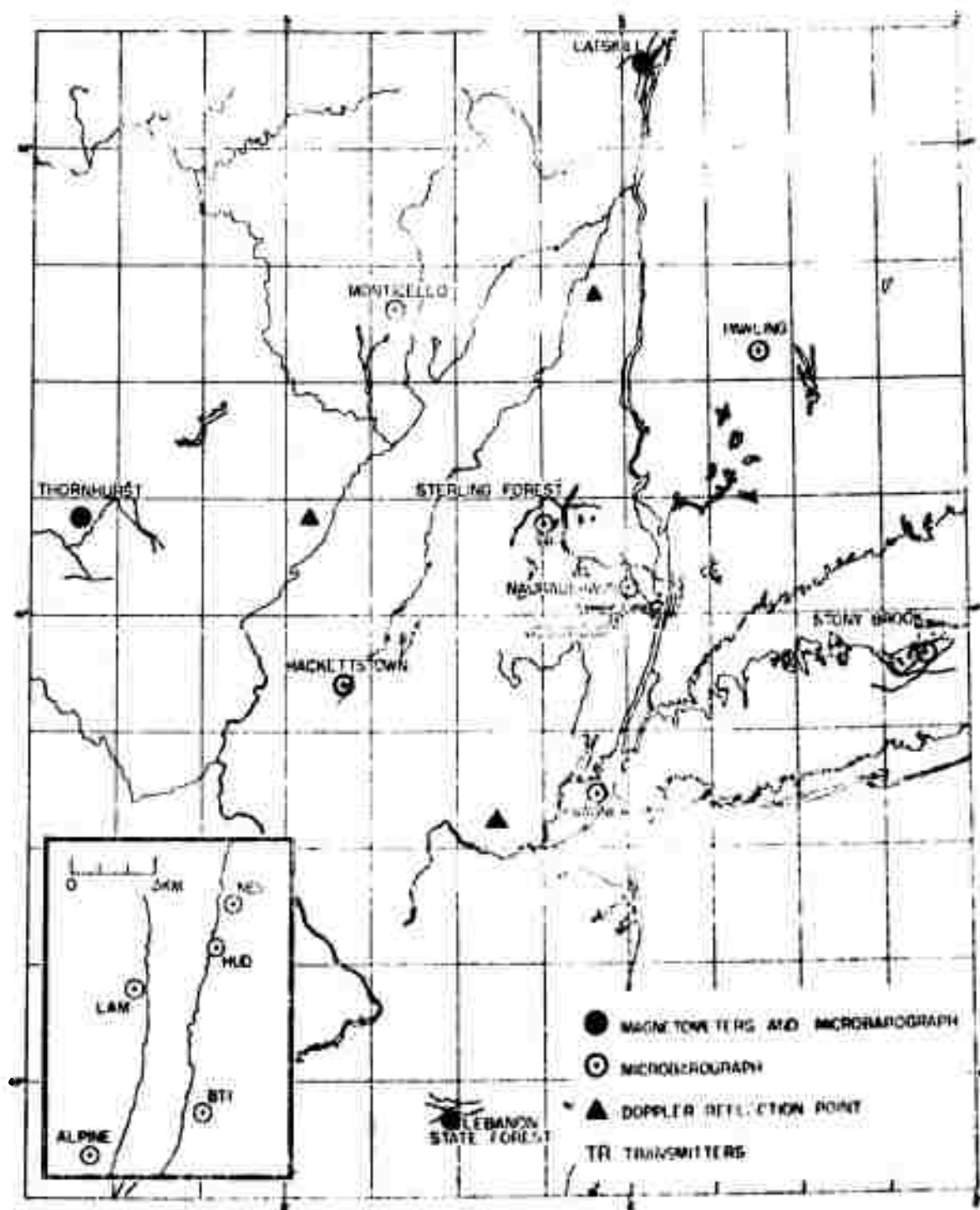


FIGURE 1. Array geometry.

to permit automatic digitization. The digitization process does not permit distinguishing between the ordinary and extraordinary modes of propagation of the sounding radio waves. However, it has been observed that, in general, the ordinary mode is the dominant one and, therefore, we have assumed that our observations are a measure of this mode. The microbarograph, magnetometer and phase-path data were filtered prior to digitization, with low pass electric filters having the appropriate time constant to minimize aliasing effects.

Sounding frequencies of 4.824 MHz and 6.030 MHz were used in the phase sounders. At these sounding frequencies and at times corresponding to the arrivals of the rocket-generated signals the radio reflection heights were between 160 km and 200 km for 4.824 MHz and between 210 km and 240 km for 6.030 MHz. These radio reflection heights were determined from true height analyses of ionograms of the Hanover Station of Dartmouth College.

During the Apollo 8 and 9 events only one or two phase sounders were in operation and phase velocity estimation for these events was not possible. At the time the signals generated by the Saturn-Apollo 10 launching were expected over the array, the phase-path and magnetometer records appear very disturbed. This appearance of the records was due to a Sudden Ionospheric Disturbance (SID) which started at 1710 UT and lasted for about two hours. The noise level on the records was so high that they were unsuitable for any type of analysis. If any signatures induced by acoustic-gravity waves were present, they were probably masked by the noise.

Doppler records for the Apollo 11 event showed a high level of noise at some of the array stations. This fact prevented the use of an array processor on the records. Nevertheless, a careful examination of the Westwood

station records which were the least noisy revealed that during the time interval 1430 to 1533 UT, doppler fluctuations showing characteristics similar to those observed during other launches can be distinguished on the traces (Fig. 2).

Microbarograph array data for several of the events have been extensively analyzed using frequency and velocity filtering as well as other types of filters and beamforming techniques. A filter that has proved to be very effective in other circumstances, which consists of cross-correlating each channel with the doppler trace containing the signal, (same principle as the matched filter) was applied to the microbarograph data. The output of this filter was then beamformed. This careful and time consuming search, however, has not been able to detect any signals or changes in character on the pressure records corresponding to those found on the doppler records.

Magnetometer recordings have been similarly analyzed, although not as extensively as in the case of the pressure fluctuation data. The results of the analyses indicated that there were no signatures in the magnetometer records at the times that they are found on the phase-path records.

Figure 2 shows the doppler records corresponding to the Westwood station. The records have been drawn in such a way that the lift-off times are lined up along a vertical line. The plotting scales are the same for all records. The data have been digitally filtered with a Fourier filter having a band pass 0.0167 Hz - 0.000333 Hz.

Visual inspection of the records in Figure 2 shows the following:

- a) At about 56 minutes after lift-off time, the character of the phase-path fluctuations changes from more or less random-type small amplitude variations into more coherent larger amplitude wave trains.

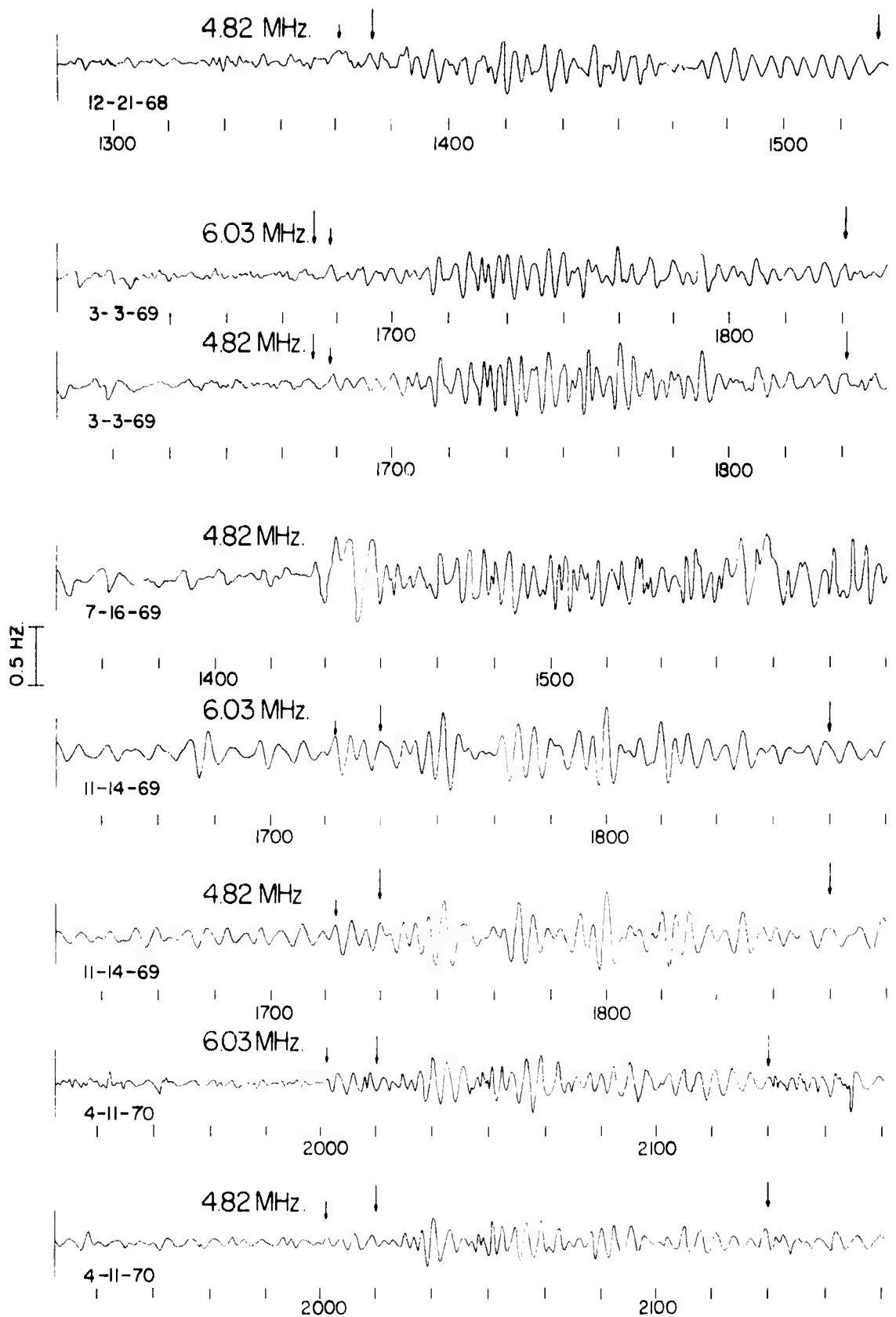


FIGURE 2. Phase-path records corresponding to Saturn Apollo launches.

- b) The amplitude and frequency content of the wave trains change with time, suggesting in some cases a beating pattern. The maximum peak-to-trough amplitudes of the wave trains are of the order of $\Delta f = 0.5$ Hz.
- c) The duration of this more organized type of motion is about 60 minutes and the periods range from about 60 seconds to 240 seconds.
- d) Another feature that can be recognized in the records is a smaller amplitude (0.2 Hz) short wave train leading the larger amplitude wave trains. This is indicated by a small arrow on top of the records on Figure 2.

Power spectral analysis using the Blackman-Tukey algorithm (Blackman & Tukey, 1959) was applied to the unfiltered data. The time interval used in the analysis is indicated by the arrows in Figure 2. The results of the analyses are shown in Figure 3; some common features can be recognized in the power spectral curves corresponding to the various events.

- a) There is a broad peak centered at 0.0056 Hz (180 sec) in both the 4.824 MHz and the 6.03 MHz data.
- b) The width of the peak is of about 3 octaves extending from 0.00333 Hz to 0.0111 Hz.
- c) The height of the peak is of the same order for all events. The peaks of the 6.03 MHz data are smaller than those of the 4.824 MHz data.

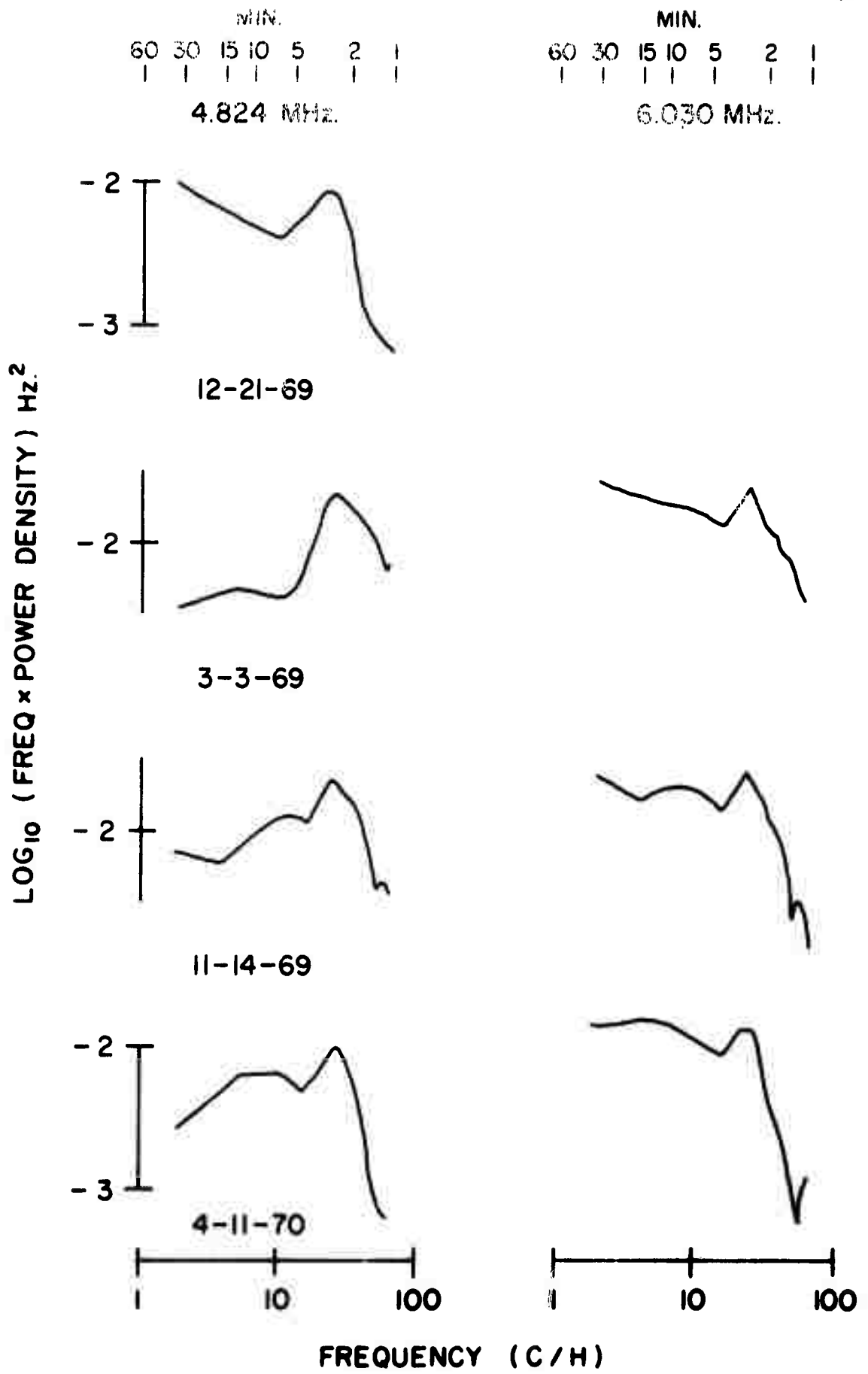
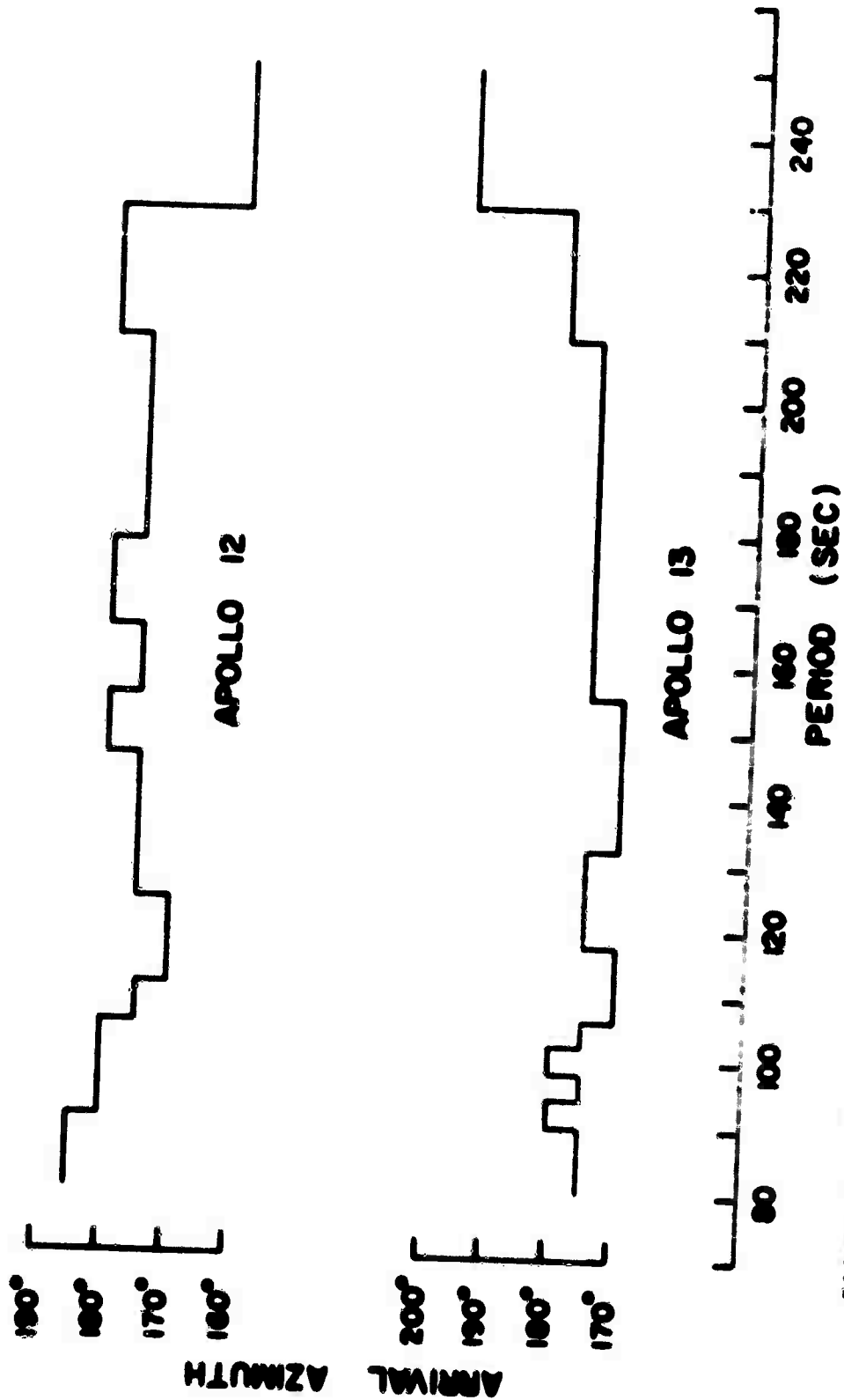


FIGURE 3 Power spectra of magnetic storm activity

Array processing of the phase-path data showed that the arrival azimuth of the signals measured at the center of the array (41.1°N , 74.12°W), varies slightly from the average azimuth of 175° as a function of period. The largest change corresponds to the longest period wave trains (Fig. 4); this shift being eastward for the Apollo 12 and westward for the Apollo 13 events respectively. The flight trajectory parameters are very similar for all Apollo launches; in particular, the altitudes of the rocket corresponding to the various arrival azimuths were approximately the same for Apollo 12 and 13 launchings. A major difference in the trajectory parameters is that the second stage center engine was powered a longer time (460 sec) during Apollo 12 than during Apollo 13 (330 sec); this means that in the first case energy was put at ionospheric levels for 130 seconds more.

Assuming that the arrivals corresponding to the various launchings, for which we do not have array data, came from the same average directions as those of Apollo 12 and 13, and using flight trajectory data to locate the source in time and space, it is possible to compute approximate values of group velocity as a function of period. This was done by applying the method of Ewing and Press (1954) to the records shown in Figure 2. The results are shown in Figure 5.

Data corresponding to large rocket launchings from the Soviet Union's launching sites at Tyuratam and Plesetsk such as Luna 15, Molniya 1L, Soyuz 6 and others were analyzed to see if some signatures could be detected. The results of these analyses showed that if the signatures were present, they were below the detection threshold. Similar results were obtained for other rockets launched from Cape Kennedy.



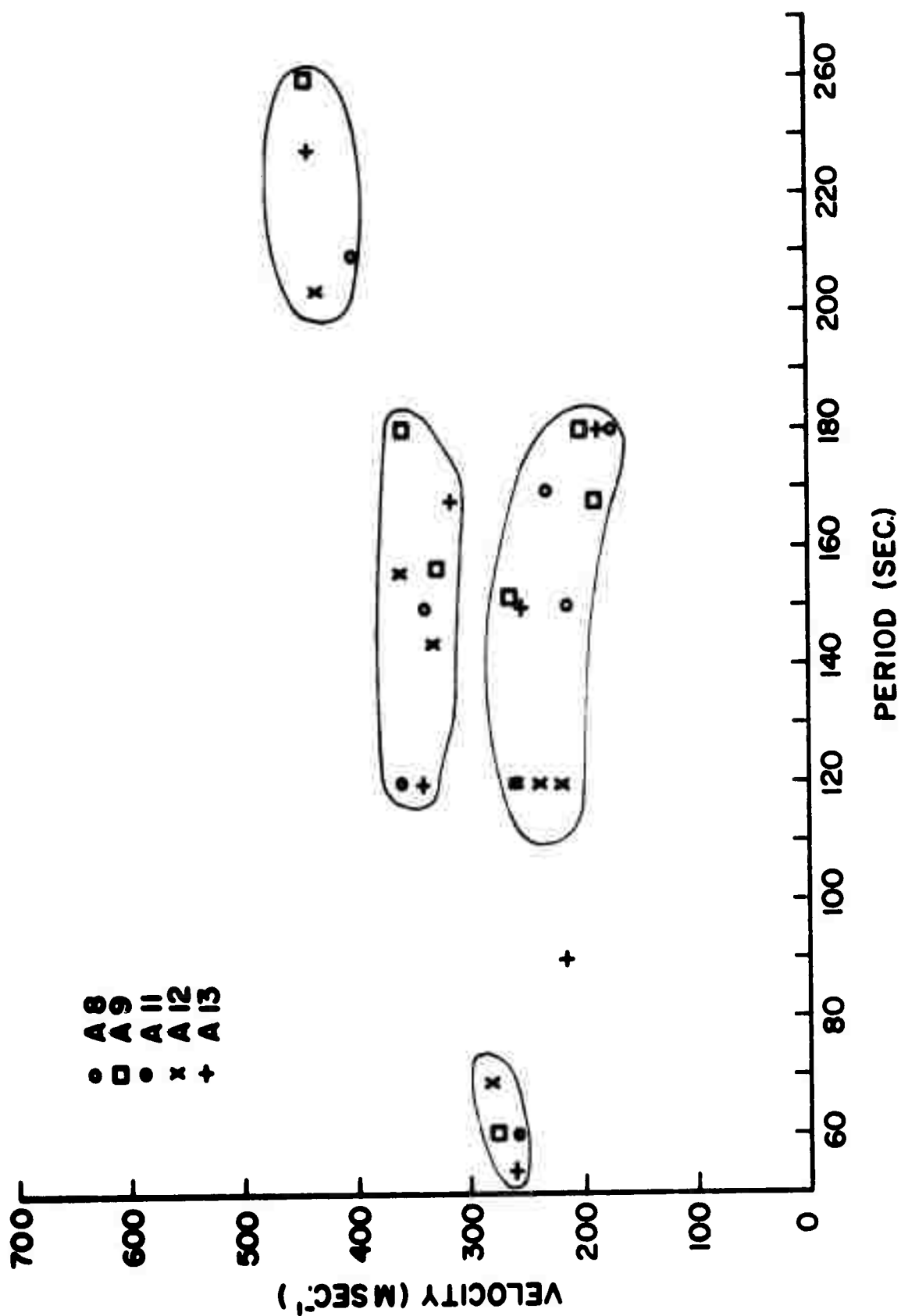


FIGURE 5. Group velocity as function of period for Saturn Apollo launches 8, 9, 11, 12 and 13.

Discussion

The fact that signatures have been identified on phase-path records corresponding to several Apollo launches confirms the hypothesis that these signatures are caused by Saturn-Apollo rockets. The average azimuth of arrival of 175° intersects the ground level projection of the trajectory at 30.74°N , 72.76°W . The altitude of the rocket at this point is 160 km. This strongly suggests that the atmospheric disturbances are generated at ionospheric heights.

The data presented here is representative of winter, spring and summer upper atmospheric conditions. The lack of amplitude and frequency content variability from event to event suggests that seasonal variation of lower atmospheric conditions does not have an important effect on the propagation of the acoustic-gravity waves that induce these phase-path signatures. This finding agrees with the ray tracing results of Balachandran and Donn (1971) and also lends support to the suggestion made by Tolstoy et al. (1970) that the propagation takes place in the upper atmosphere.

The failure to detect signals from other rockets launched from Cape Kennedy and rockets launched at greater distances was probably due to longitudinal attenuation of the acoustic waves or to the fact that the energy input into the atmosphere at ionospheric levels was too small to generate these long period waves.

Visual examination of phase-path sounder records for stations located in the Mid-western United States indicates that no signatures were present at the time of the Apollo 12 event (Davies, 1970), and Apollo 15 (Lerfald, 1971). This lack of arrivals could be due to attenuation since the distance from the

source to these stations is more than twice the distance from the source to Westwood. However, it is also possible that this is due to the better response of the ionosphere to neutral atmospheric waves traveling in an almost North-South path (Hooke, 1970) as is our case.

The difference in arrival azimuth for the longer period signals during Apollo 12 and 13 events could be due to the longer duration of the powered stage on Apollo 12; since this longer duration made the rocket exhaust plume travel more towards the east than it did on the Apollo 13 event. Why the longer periods are the most affected by this situation is something we cannot answer at this time. In order to answer this question one would have to solve the moving source problem. This is complicated because the component of the source velocity in the direction of the array results in appreciable doppler shifting of the source spectrum, while the vertical component of motion introduces a modulation of each mode. The latter effect is probably small since the change in altitude of the rocket during the generation of the waves is not too large.

Figure 5 shows that the group-velocity points are grouped into four separate groups. The group whose velocity and period ranges are $450 > U > 400$ and $240 > T > 200$ respectively has been identified by Tolstoy et al. (1970) as a mode whose energy is trapped in the lower thermosphere. The other groups probably belong to modes which are trapped in the upper sound channel.

Finally, it should be pointed out that absence of signatures from smaller rockets and the wavelengths involved in these observations strongly suggest that the acoustic-gravity waves are generated by the rocket exhaust rather than by a ballistic mechanism.

3. DISPERSION STUDY OF THE TID EVENT OF 23 NOVEMBER 1969

Introduction

The earth's atmosphere is a very complex medium and there are severe gaps in the knowledge of atmospheric parameters and structure, in particular for altitudes above 100 km.

In the lower 100 km of the atmosphere the structure is reasonably well known, and although the temperature and winds vary with season and geographical location it is possible, in principle, to include these effects in propagation studies (Pierce, 1966; Balachandran, 1970). However, it has been observed that following atmospheric nuclear explosions, long period (2-5 min) acoustic waves and internal gravity waves propagate at heights above 200 km (Baker and Davies, 1968; Obayashi, 1963; Herron and Montes, 1970). These observations indicate that appreciable wave energy in the acoustic and gravity wave frequency range corresponding to periods longer than a minute can be channeled at mesopause and thermosphere altitudes.

It is well known that in wave systems generated by atmospheric explosions the distribution of mode energy with height depends on the atmospheric neutral temperature structure.

Calculations of yield and height of burst based on mode analysis are at the present time marginal because of the variation of the temperature structure. Tolstoy and Lau (1970) have shown that any prediction of the vertical amplitude distribution of displacement $\xi(z)$, is prevented by the variability of the neutral structure since the displacement $\xi(z)$ is heavily dependent on the Brunt-Väisälä $N(z)$ and acoustic cutoff $\omega_c(z)$ frequencies.

A study of natural ionospheric background notions made by Tolstoy

and Montes (1971) has shown that it is possible to derive, in situ, long term averages of the buoyant resonance frequency (Väisälä frequency) of the neutral gas. This is, of course, far from the precise determination of $N(z)$ and $\omega_0(z)$ that one would like to have.

Tolstoy (1970) has pointed out that dispersion studies of Traveling Ionospheric Disturbances (TIDs) could provide some information about the neutral structure at ionospheric heights.

During the operation of phase-path sounder array by Teledyne Isotopes several TIDs were observed. Their general properties have been reported by Rao (1970). In order to test Tolstoy's suggestion, a TID was selected and subjected to dispersion analysis. The results of this study are reported here.

Data and Method of Analysis

The TID event selected for this study is shown in Figure 6. At the time of the event there were five doppler sounders in operation. The four sounders forming the array operated at a sounding frequency of 4.824 MHz; the fifth sounder operated at Westwood using a sounding frequency of 2.412 MHz. The magnetograms corresponding to the time interval of the TID event showed very little activity. This fact can be taken as an indication that the disturbance observed on the phase-path records had a hydrodynamic origin, i.e., it was induced by an atmospheric gravity wave.

The starting time of the event was set at 2000 UT on November 23, 1969. However, it is more difficult to estimate the ending time of the TID. If too long a record is included in the analysis, the doppler fluctuations which are not connected with the TID will make the coherency drop and will introduce errors in the azimuth and phase velocity estimates. On the other hand,

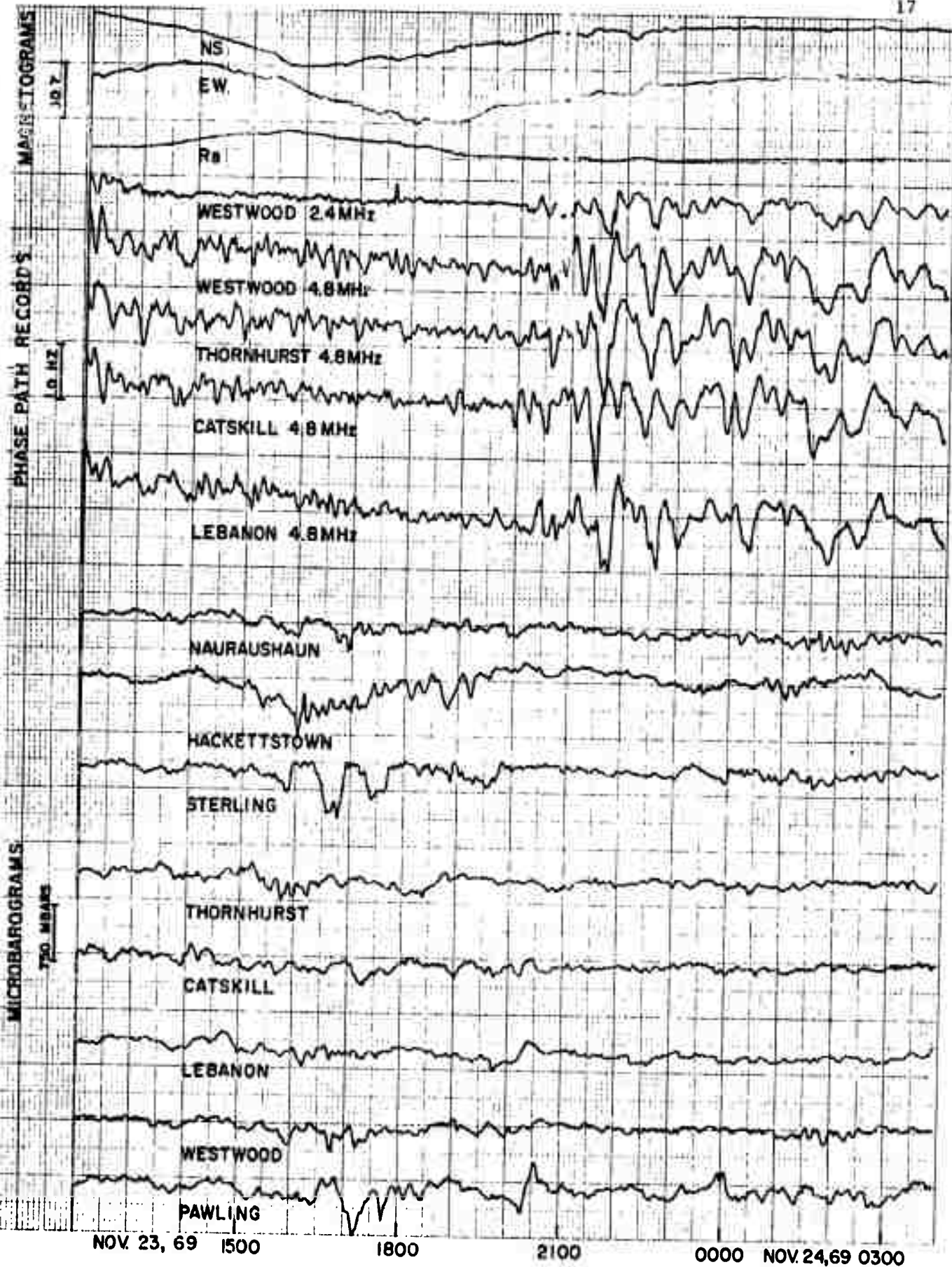


FIGURE 6. TID event of 23 November 1969.

if the length of the record chosen is too short, one could be taking out of the analysis important contributions to the TID and the frequency resolution of the analysis will suffer. With the help of cross spectral analyses the ending time for the event was chosen to be at 0130 UT on 24 November 1969.

The dispersion analysis of the phase-path records at the time interval 2000 UT - 0130 UT was carried out using the cophase analysis described by Posmentier and Hermann (1971). The results of the analysis have been plotted as points on the ω -k plane (Fig. 7). Two conclusions can be drawn from this figure:

- a) the points fall on the phase velocity range 120 to 240 m sec⁻¹
- b) they probably belong to various modes.

The relative phases as function of frequency between the 2.412 MHz data and the 4.824 MHz at Westwood were found from the cross spectral analysis of these two records. The reflection heights corresponding to the two sounding frequencies were obtained from the true height analysis of the Hanover station of Dartmouth College.* The average reflection heights were found to be 248 km for the 4.824 MHz and 233 km for 2.412 MHz respectively.

Discussion

The points on Figure 7 can not be joined arbitrarily. A propagation model must be constructed assuming reasonable values for the parameters until a fit to the data can be obtained.

* Made available to us through the kindness of Drs. M. G. Morgan and C. Calderon of the Radio Physics Laboratory.

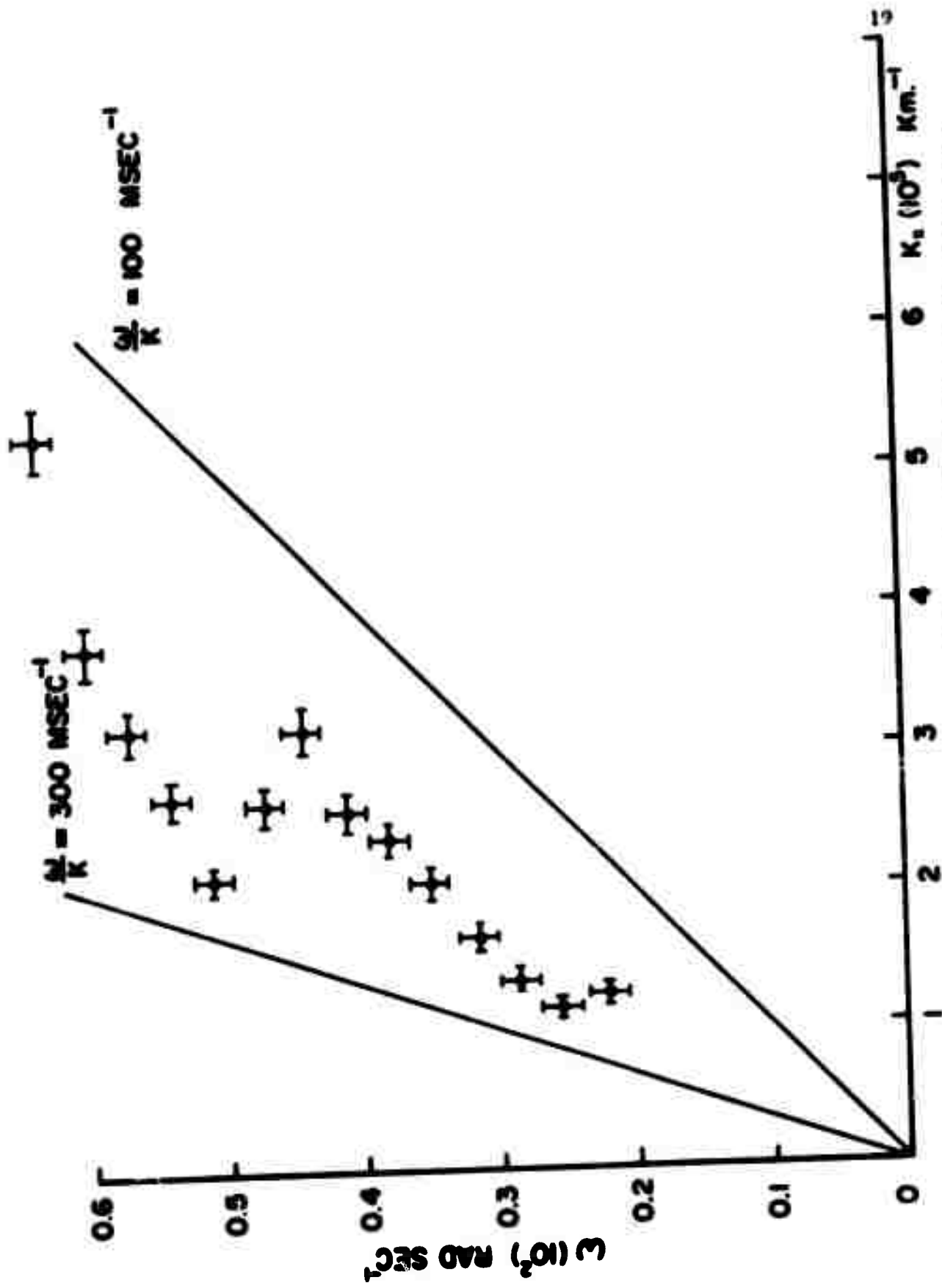


FIGURE 7. --A plot showing results of dispersion analysis of the TID of Figure 6.

Tolstoy and Lau (1970) have shown that the region of the atmosphere where $\omega_0 < N$ must be taken into account when studying the propagation of atmospheric waves is above 100 km. Following these authors, we write the solution of the differential equation for the pressure as

$$p = \rho^{-1/2} q \exp [(ax - \omega t)] \quad (1)$$

where

ρ is the neutral gas density,

a is the horizontal wave number,

ω is the angular frequency of the disturbance and

q satisfies the Helmholtz equation

$$q_{zz} + \beta^2 q = 0 \quad ; \quad (2)$$

the sub-index z indicates differentiation with respect to that variable and β is a function of z and is given by

$$\beta^2 = \frac{1}{C^2} (\omega^2 - \omega_{op}^2) + \frac{1}{V_x^2} (N^2 - \omega^2) \quad (3)$$

with

$$V_x = \frac{\omega}{a},$$

$$\omega_{op}^2 = C^2 \left\{ \frac{1}{4} g^2 \frac{z^2}{C^4} + \frac{1}{4} \left[\frac{d}{dz} (\text{Ln} C^2) \right]^2 + \frac{1}{2} \frac{d^2}{dz^2} (\text{Ln} C^2) \right\} \quad (4)$$

and

$$N^2 = C^2 \left\{ \left[\frac{g^2}{C^4} (z-1) + \frac{g}{C^2} \frac{d}{dz} (\text{Ln} C^2) \right] \right\} \quad (5)$$

where C is the speed of sound, g the gravity acceleration and τ the ratio of specific heats is taken to be 1.4.

The variation of the speed of sound with z above certain height can be approximated by assuming a law for $C(z)$ of the form.

$$C^2(z) = Kz + C_0^2, \quad (6)$$

where C_0 is the sound velocity at $z = 0$, and K is a parameter to be determined once the atmospheric model is assumed.

From equations (4), (5) and (6) we obtain ω_{op} and N in function of the new parameter K

$$\omega_{op}^2(z) = \frac{1}{4C^2} (g^2\tau^2 - K^2) \quad (7)$$

and

$$N^2(z) = \frac{g}{C^2} (K + 0.4g). \quad (8)$$

If now the atmospheric structure is approximated by taking the origin of coordinates at $h = 120$ km, the parameter K can be determined from equation (6) using values for C_0 and $C(140)$ from the U.S. Standard Atmosphere tables.

$$\begin{aligned} C_0 &= 3.92 \times 10^2 \text{ m sec}^{-1} & z &= 0 \\ C &= 9.90 \times 10^2 \text{ m sec}^{-1} & z &= 1.40 \times 10^5 \text{ m} \end{aligned}$$

This gives

$$K = 5.92 \text{ m sec}^{-2}, \quad (9)$$

and

$$\left. \begin{aligned} N &= 2.40 \times 10^{-2} \text{ rad sec}^{-1} \\ \omega_{op} &= 1.46 \times 10^{-2} \text{ rad sec}^{-1} \end{aligned} \right\} \text{ at } z = 0 \quad (10)$$

$$\left. \begin{aligned} N &= 1.18 \times 10^{-2} \text{ rad sec}^{-1} \\ \omega_{\text{op}} &= 0.57 \times 10^{-2} \text{ rad sec}^{-1} \end{aligned} \right\} \text{ at } z = 1.40 \times 10^5 \text{ m} \quad (11)$$

Since the observations presented here belong to altitudes of 220 - 250 km, and we are not concerned with amplitude calculations at lower levels, we do not have to worry about the structure below 120 km and this can be approximated in the form shown in Figure 8.

Using equations (3), (6), (7) and (8) with (9), values of β were calculated for the various points of the ω -k plot (Fig. 9).

It has been pointed out in the infrasonic literature that when interpreting observations of TIDs special care should be taken to see if these observations belong to a turning point (Tolstoy, 1971). It is well known that WKB approximations are not valid at turning points, and the interpretation of the ω -k data should be done taking into consideration this fact. To test our data for proximity to a turning point we have used the criteria given by Tolstoy (1971).

$$\text{If } \frac{\omega}{\beta} \approx \frac{\omega}{\frac{\Delta\phi}{\Delta z}} \text{ far from a turning point,} \quad (12)$$

and

$$\text{if } \frac{\omega}{\beta} \gg \frac{\omega}{\frac{\Delta\phi}{\Delta z}} \text{ near a turning point,} \quad (13)$$

where $\Delta\phi$ is the phase difference between the observations made at two different levels and Δz is the vertical distance between these levels. The results of the cross-spectral analysis between the 4.824 and 2.412 MHz data showed that the phase differences are

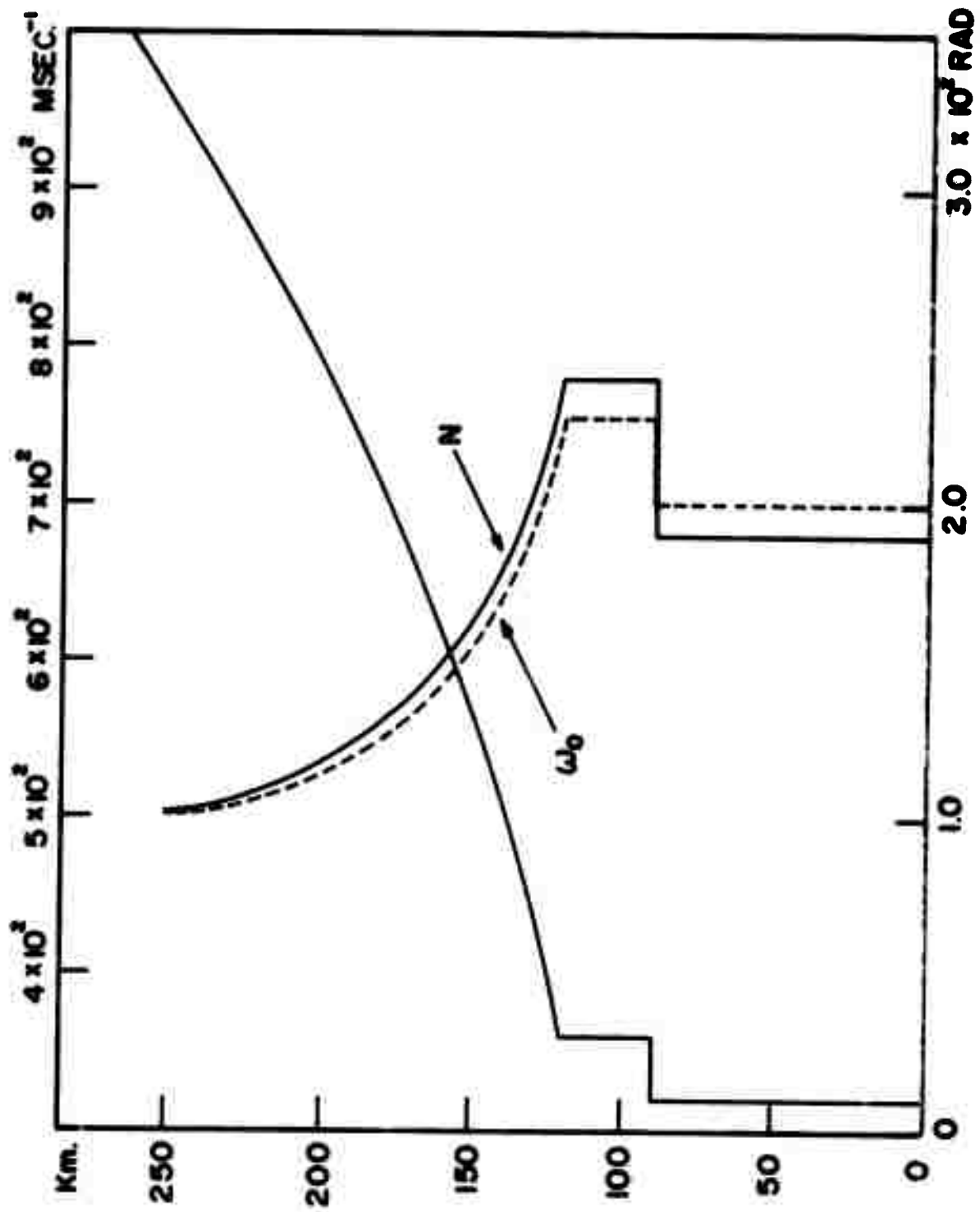


FIGURE 8. Atmospheric model used in the analyses of the TID of Figure 5.

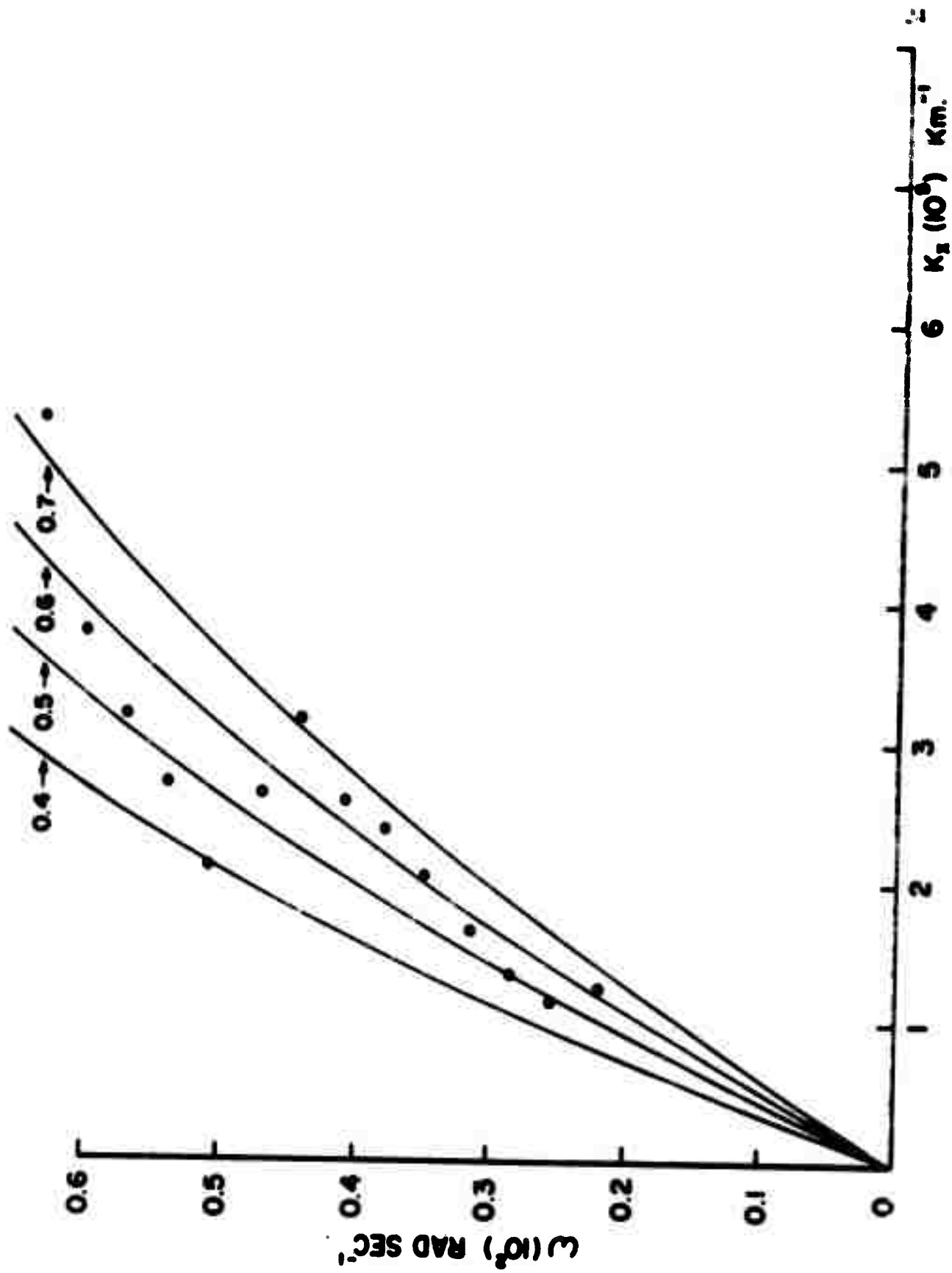


FIGURE 9. ω - K_2 curves for a height of 220 km using atmospheric model of Figure 4.

$$\Delta\theta = 51.5 \pm 3^\circ \text{ for } \omega = 0.002221 \text{ rad sec}^{-1}$$

$$\Delta\theta = 51.0 \pm 4^\circ \quad \omega = 0.0031733 \text{ rad sec}^{-1}$$

$$\Delta\theta = 52.2 \pm 7^\circ \quad \omega = 0.004413 \text{ rad sec}^{-1}$$

The corresponding values of $\omega/(\Delta\theta/\Delta z)$ were 37.0 m sec^{-1} , 53.5 m sec^{-1} and 73.2 m sec^{-1} respectively. The values of ω/δ obtained for the same angular frequencies were 29.0 m sec^{-1} , 45.0 m sec^{-1} and 50.37 m sec^{-1} respectively. It can be seen that taking into account the experimental errors, the data shown here satisfies condition (12) indicating that our observations have been made far from a turning point.

A refined value of δ can be obtained if we set $\delta = \Delta\theta/\Delta z$ and use this adjusted value of δ to recompute N and ω . After this was done ω - k curves using δ as a parameter were computed and the results are shown in Figure 9. A study of this figure indicates that our earlier conclusion about the points on the ω - k plane was correct, i.e., the points correspond to various modes.

It has been illustrated here that it is possible to derive structure information from the dispersion of naturally occurring TIDs. Continuation of this type of study would secure more data about the neutral temperature structure at ionospheric altitudes.

4. PARAMETRIC MODEL OF IONOSPHERIC BACKGROUND MOTIONS

Current studies of atmospheric acoustic gravity wave propagation have shown that there is a wide distribution of the energy of the various modes throughout the atmosphere, including the ionosphere. Consequently, it was natural to think that observations of ionospheric motions produced by motions of the neutral component of the atmosphere could be used to detect acoustic-gravity wave modes generated by nuclear explosions and propagating at ionospheric heights. Disturbances associated with nuclear explosions in the atmosphere have indeed been recorded on both ground level microbarographs (Donn and Shaw, 1967; Tolstoy and Herron, 1970) and at ionospheric heights (Obayashi, 1963; Baker and Davies, 1968; Herron and Montes, 1970).

A simple method of monitoring ionospheric motions is to measure the variations of the phase-path of an ionospherically reflected H.F. radio wave (Davies, 1962). In 1969, with this objective in mind, Teledyne Isotopes set up a four element phase-path sounder array.

It is well known that in order to describe the performance of an array, the knowledge of the background characteristics of the parameter to be measured is essential.

Power spectral analysis is often used to investigate the characteristics of the background. The power spectrum of the phase-path variations is a function of several parameters. It reflects, in particular, the spectral characteristics of the disturbance sources, the manner in which this mechanism is coupled into the medium, the efficiency of coupling to changes in the electron density, and the structure of the medium.

Ionospheric background motions are known to vary over time (Georges, 1967; Tolstoy and Montes, 1971). This variation can be appreciable at times, with large changes occurring over a period of several hours. Computation of the long-term average spectrum indicates how the power is distributed as a function of frequency but is of little use for predicting the background motion for any particular time of day. The objective of this work was to make a realistic and physically compatible parametric model for the power spectrum of the phase-path fluctuations in which the variation of the parameters over time accounts for the nonstationarity of the background motion.

Data and Method of Analysis

During the years 1969 and 1970, Teledyne Isotopes recorded micro-barographic, geomagnetic and phase-path fluctuation data in the New Jersey area as part of a program to study atmospheric background motions. The set of data used in this study corresponds to the month of June 1969 (Fig. 10), and it was chosen because of the author's familiarity with it from a previous study. Since our main concern is the phase-path (doppler) sounder data, these will be described briefly. The sounder operated at almost vertical incidence (separation between transmitter and receiver was less than 30 km), with a sounding frequency of 4.82 MHz; at this frequency the ionosonde was effectively sampling the F-region of the ionosphere. The data were automatically digitized after being filtered with a 120 second low pass filter, using a sampling interval of 60 seconds, in this way minimizing aliasing effects.

The digitizing process does not distinguish between the ordinary and extraordinary modes of propagation of the sounding radio wave; since it is

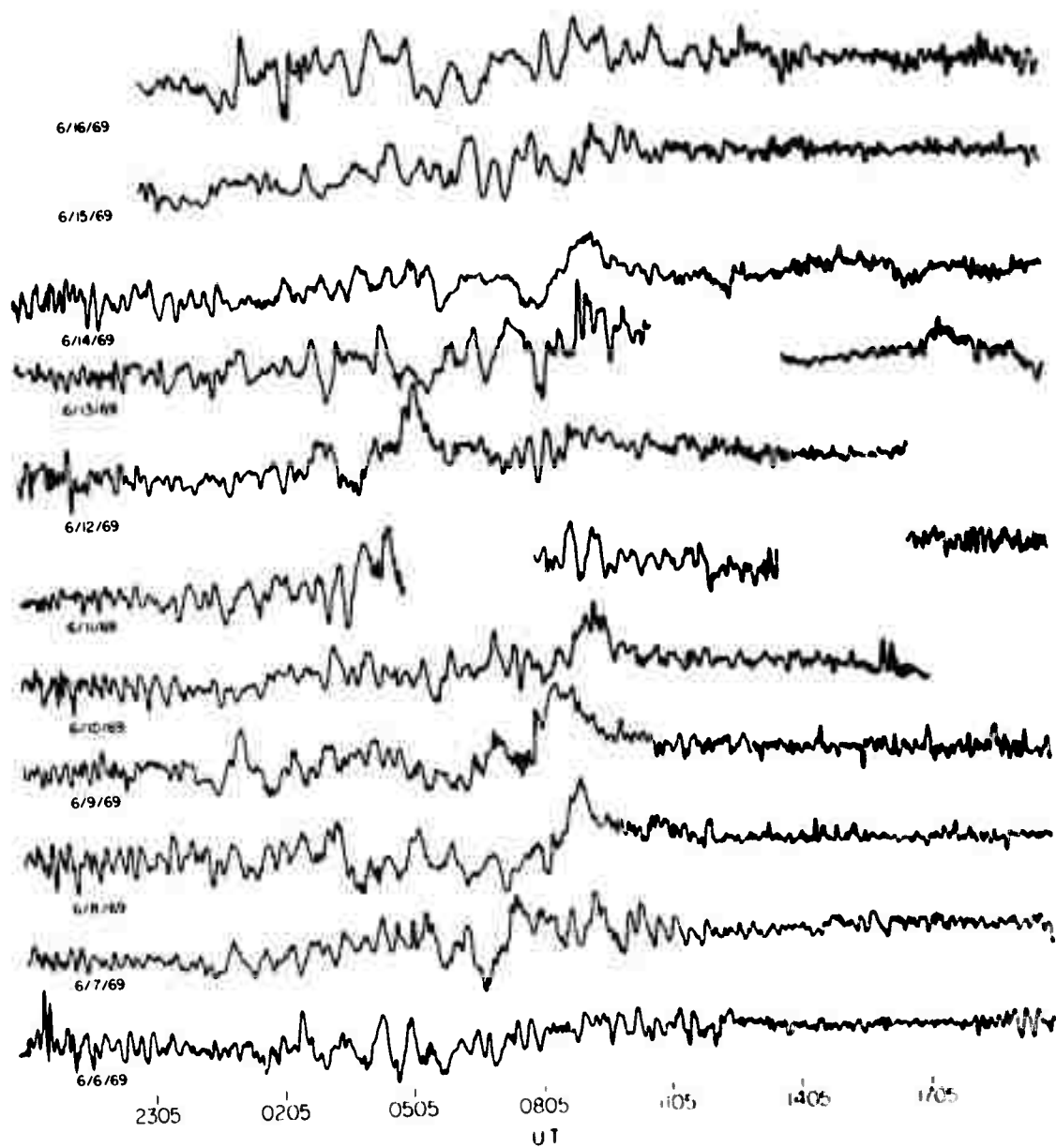


FIGURE 10a. Phase-path fluctuation data used in the parametric model.

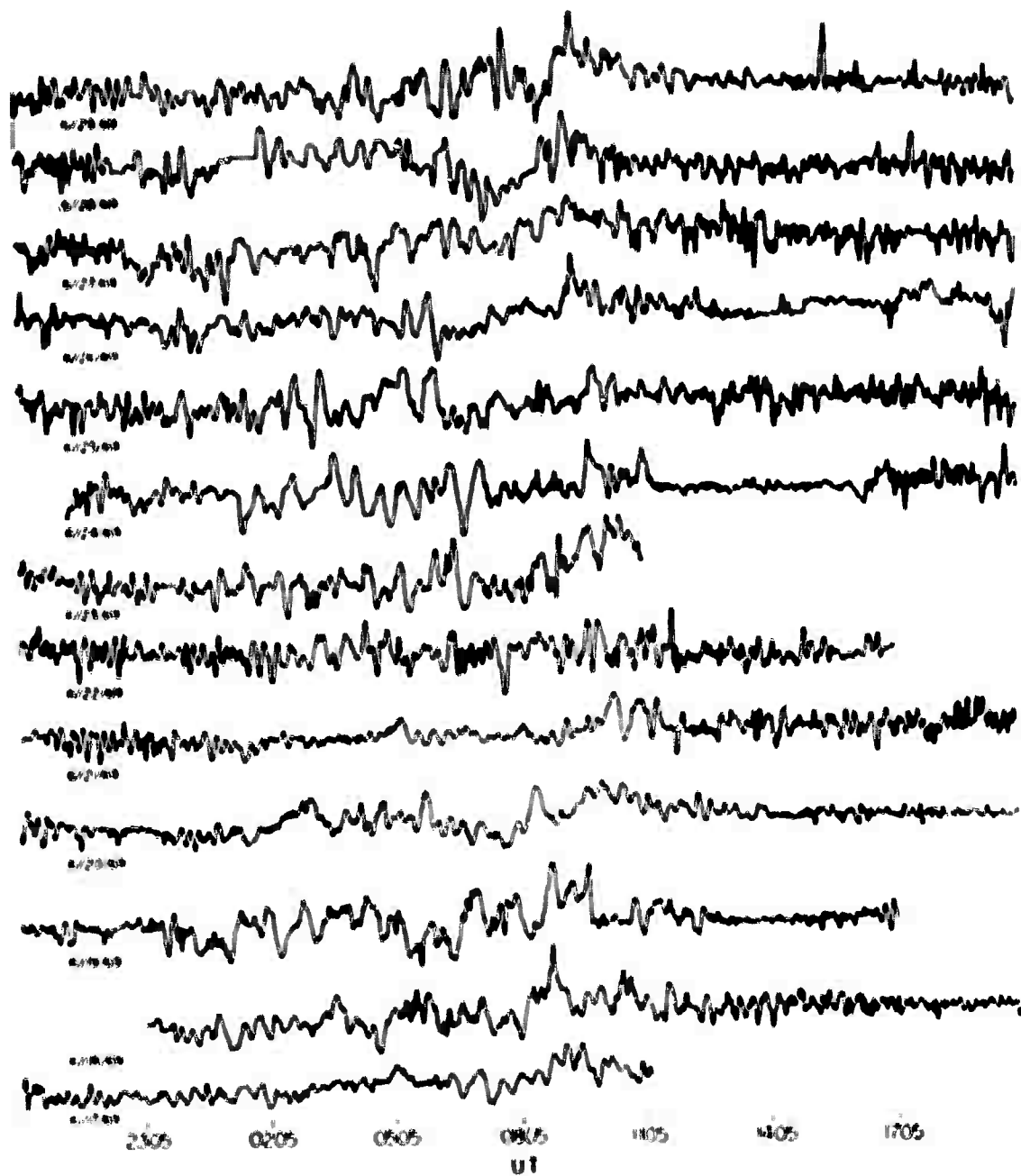


FIGURE 10b. Continuation of Figure 10a.

commonly observed that the ordinary mode overshadows the extraordinary one, we have assumed that our data provides a measure of the ordinary mode.

The data sample representing 648 hours (27 days) was subjected to power spectral analysis, and the Log_{10} of the power spectral density was computed for every three-hour interval using the Blackman-Tukey algorithm (Blackman and Tukey, 1959). Special care was taken to remove from the analyses those time intervals corresponding to solar flares or magnetic storms since these events are not part of the background.

In order to test the effect of sampling on the analysis, 2, 3, 4 and 6-hour time intervals were used in the computation of the power spectra for a short data sample (240 hours). It was found that, in general, the length of the time interval did not greatly affect the amount of variance that can be explained by the model. However, taking into account aliasing problems and the desired frequency resolution, it was a natural compromise to arrive at the three-hour sampling intervals used throughout the analysis.

The frequency band covered by the spectral analysis was divided into 13 elementary frequency bands of equal bandwidth $\Delta f = 0.0002778$ Hz. These elementary frequency bands correspond to the center frequencies 1.0, 2.0, 3.0,13.0 CPH, that is, periods of 60, 30, 20,4.61 minutes, respectively. Typical power spectra are shown in Figure 11.

The results of the power spectral analysis indicate that the ionospheric background motions constitute a nonstationary process, i.e., the power and the spectral shape of the phase-path fluctuations vary over time (Fig. 12).

The average shape of the phase-path fluctuation spectrum suggests that a linear approximation of the log spectrum could be used as a model. The

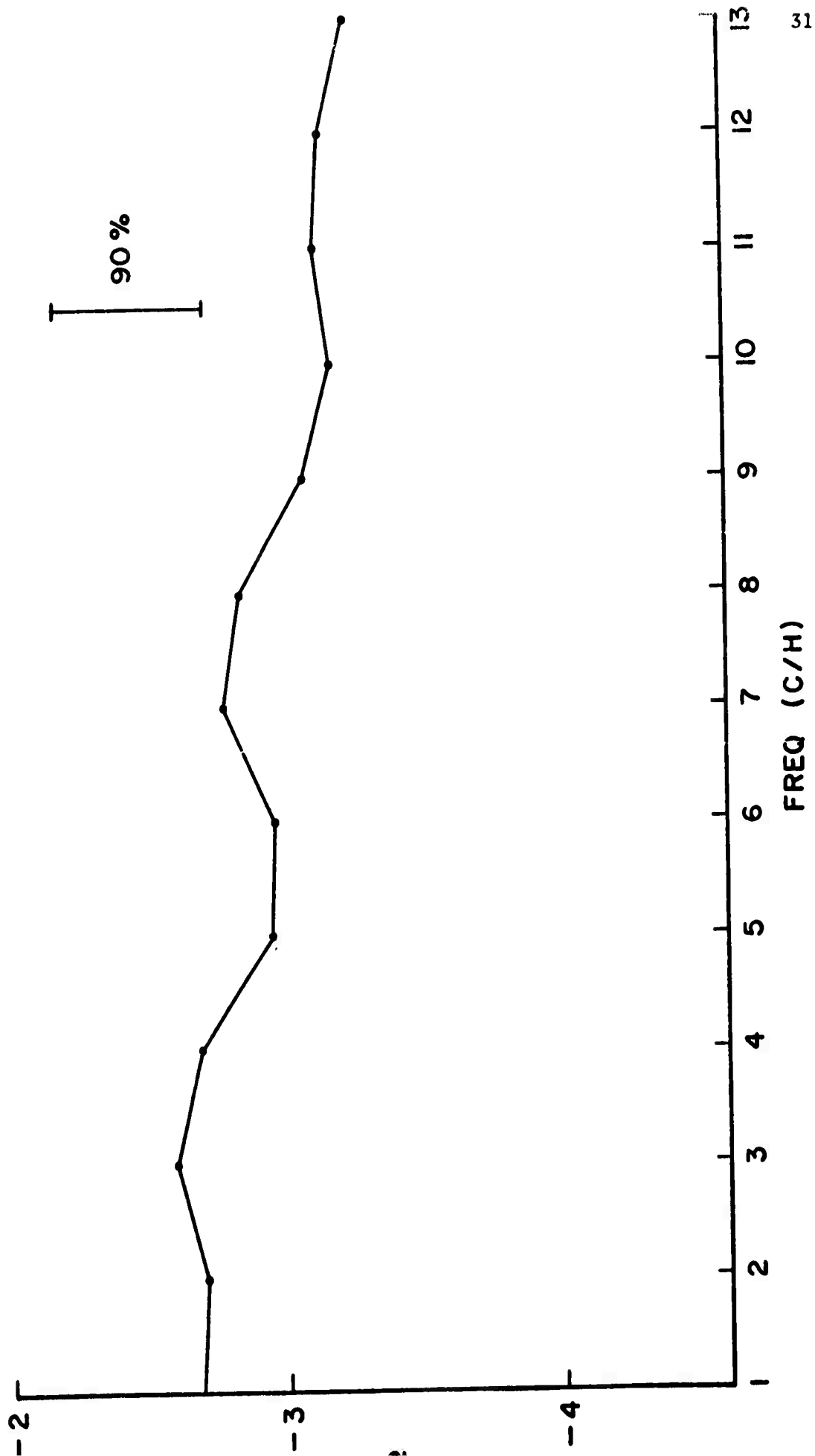
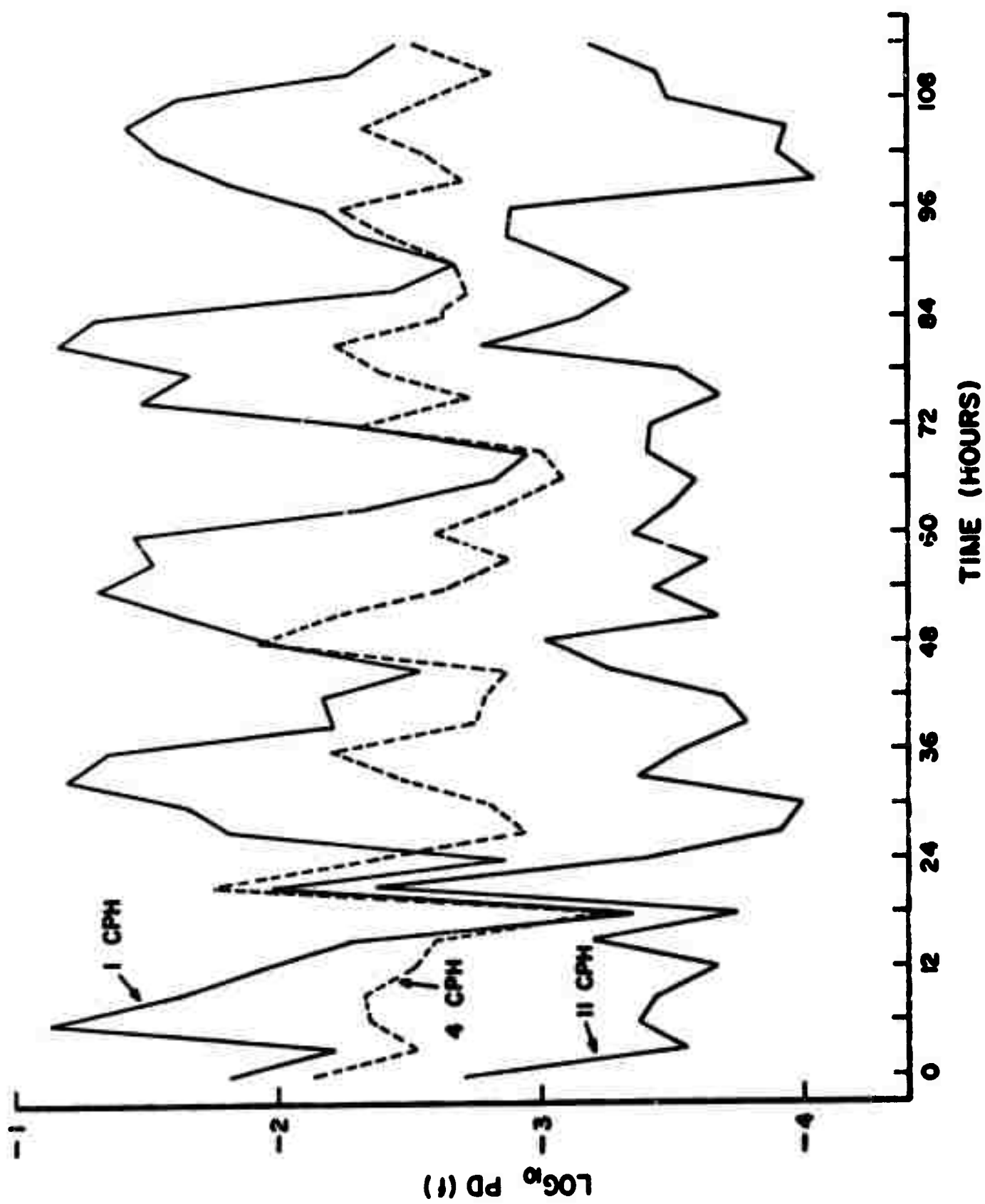


FIGURE 11. Sample power spectrum.



statistics of such type of approximation have been given by Hinich (1967).

Let $\hat{S}_{ij}(f_k)$ denote the average power in the band of center frequency f_k computed from the j^{th} 3-hour long record on the i^{th} day ($i=1, \dots, n$; $j=1, 2, \dots, 8$; and $k=1, 2, \dots, 13$) and let $S_j(f_k)$ denote the true power, i.e., the long term average value of $\hat{S}_{ij}(f_k)$ is $S_j(f_k)$. It is well known that the variance of $\text{Log}_{10} \hat{S}_{ij}(f_k)$ is the same for all frequencies f_k (Hinich and Clay, 1968). The sampling variability of $\text{Log}_{10} \hat{S}_{ij}(f_k)$ about the true value $\text{Log}_{10} S_j(f_k)$ depends only on the record length and on the bandwidth.

Suppose that we make the following linear model for $\text{Log}_{10} S_j(f_k)$:

$$\text{Log}_{10} S_j(f_k) = a + \beta_j f_k + F_j$$

where a is a constant effect, β_j is the slope of $\text{Log}_{10} S_j(f_k)$ as a function of f_k for a fixed j , i.e., we are assuming that at any given time of day the log spectrum of the background is linear in frequency. F_j is the effect due to the time variation. This linear model yields the following expression for $\text{Log}_{10} \hat{S}_{ij}(f_k)$: For $i = 1, \dots, n$

$$\text{Log}_{10} \hat{S}_{ij}(f_k) = a + \beta_j f_k + F_j + c_i(f_k)$$

where $c_i(f_k)$ is a zero mean Gaussian error whose variance depends only on k .

It is clear that for the model to have validity, the time variation effect must be modeled taking into account actual physical situations.

During the last two decades considerable effort has been put in understanding the upper atmosphere dynamics. It is in general agreement among upper atmospheric workers (Bolgiano, 1959), that although turbulence is a common

occurrence in the atmosphere, at least to an altitude of 100 km, the intensity of fully developed turbulence may be no more than 1 or 2 percent of the total energy; the bulk of the energy probably residing in more organized type motions such as random gravity waves, thermal winds, and tidal oscillations. Lindzen (1969) pointed out that the most significant atmospheric tides are those excited by the absorption of insolation by water vapor and ozone. These tides have periodicities of 12 and 24 solar hours. In the upper atmosphere the picture is more complicated because of the presence of ionization.

Simplified theory of ionospheric layer formation predicts vertical motion of the ionosphere as a function of solar angle (Rishbeth, 1963). Thus, we have assumed as a starting point for modeling the time variation effect, a solar control of the ionospheric motions. Sinusoidal functions, with argument of the form $\omega t = \frac{2\pi}{T} \cdot t$, were generated. The parameter t corresponds to the time at the center of each 3-hour window of the power spectral analysis, and T is a multiple or submultiple of 24 hours.

In order to investigate the relationships between the sinusoidal time functions and the Log_{10} power spectrum for each frequency band, multiple linear regression analysis was used. The sinusoidal functions entered the analysis as independent variables, while the Log_{10} power spectrum was the dependent variable. Means, standard deviations, simple and multiple correlation coefficients, etc. were calculated.

Results

The multiple regression analysis showed that the stronger contributions to the time variation of the power spectra come from those periodic functions with angular frequencies $\omega = 2\pi \text{ rad day}^{-1}$ and $\omega = 4\pi \text{ rad day}^{-1}$. Figure 13

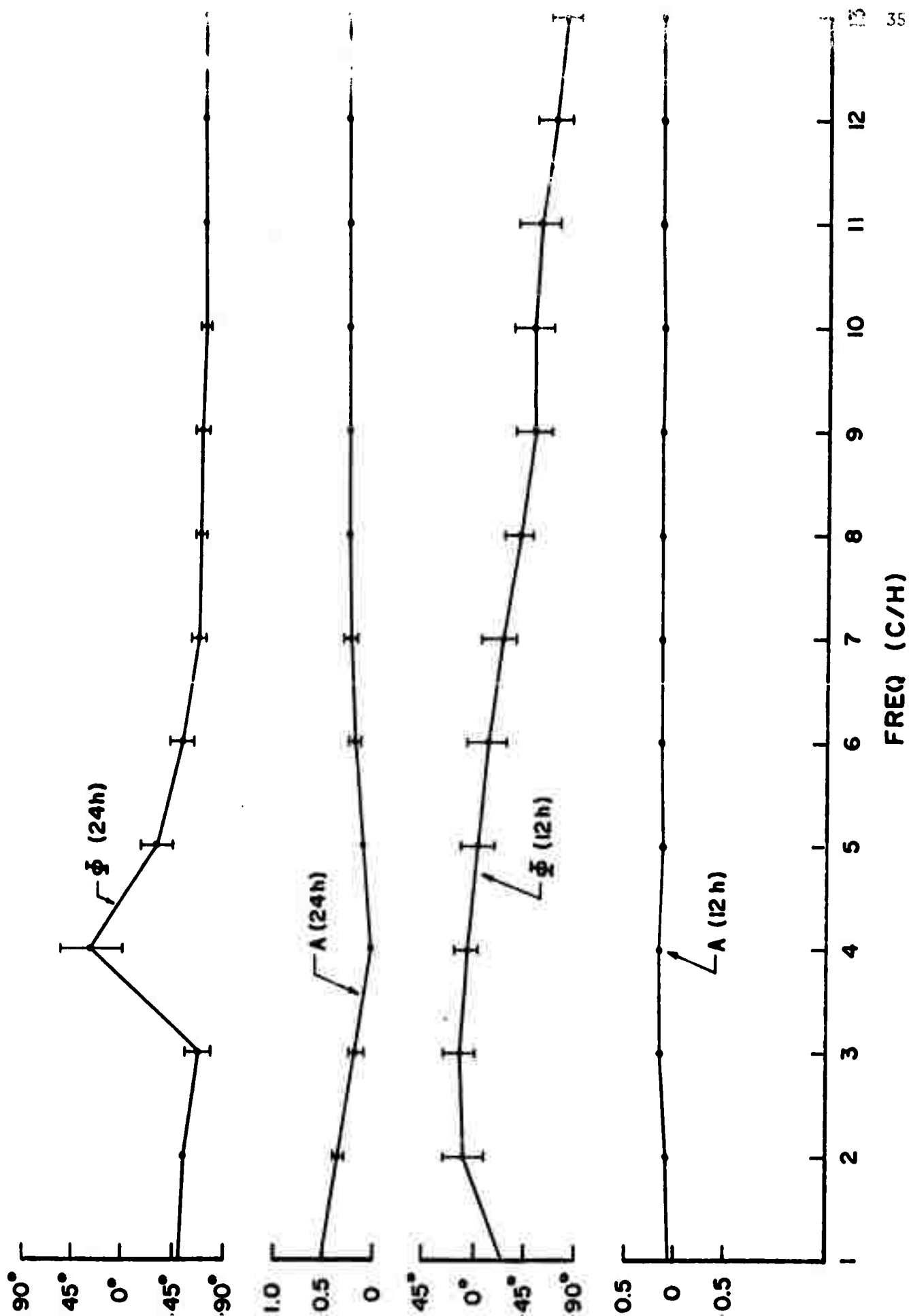


FIGURE 13. Amplitude and phase for 12- and 24-hour components of time variation as function of frequency.

shows the distribution of amplitudes and phases, as function of spectral frequency, for these functions. It can be seen in this figure that both the amplitude and the phase of the 24-hour component ($\omega = 2\pi \text{ rad day}^{-1}$) are approximately constant for frequencies higher than 6CPH. A large change in the phase between 3 and 5 CPH is also observed. The amplitude of the 12-hour component is rather constant from 2 to 13 CPH. The amplitudes of the other periodic components are smaller by an order of magnitude.

The average slope of $\text{Log}_{10} S_j(f_k)$ is given by the slope of the intercepts. Figure 14 which is a plot of the intercepts as function of frequency shows that the slope changes at a frequency of 7.25 CPH. The values of β_j corresponding to these two branches are

$$\begin{aligned} \beta_i &= -0.188 & f_k &\leq 7.25 \text{ CPH} \\ \beta_j &= -0.066 & f_k &> 7.25 \text{ CPH} . \end{aligned}$$

Examination of the multiple correlation coefficients reveals that the model can explain a good part of the time variation of the power spectra for the frequencies

$$f_k < 3 \text{ CPH}$$

and

$$f_k > 6 \text{ CPH:}$$

i.e., between 36% and 50% of the variance can be explained at these frequencies.

The distribution of the residuals for each frequency band is shown in Figure 15 in the form of smoothed histograms. The normal distribution curve is also plotted in each case.

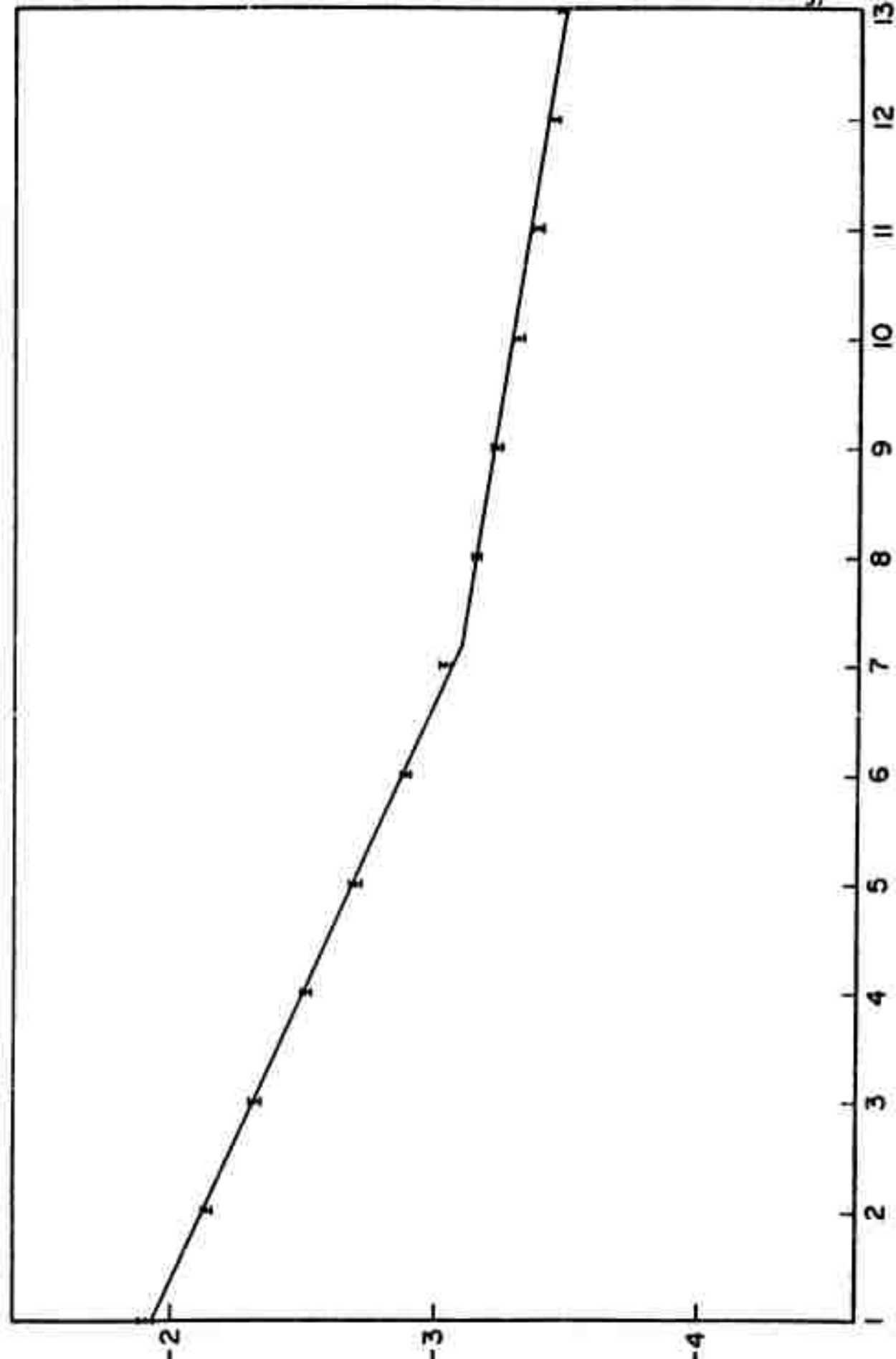
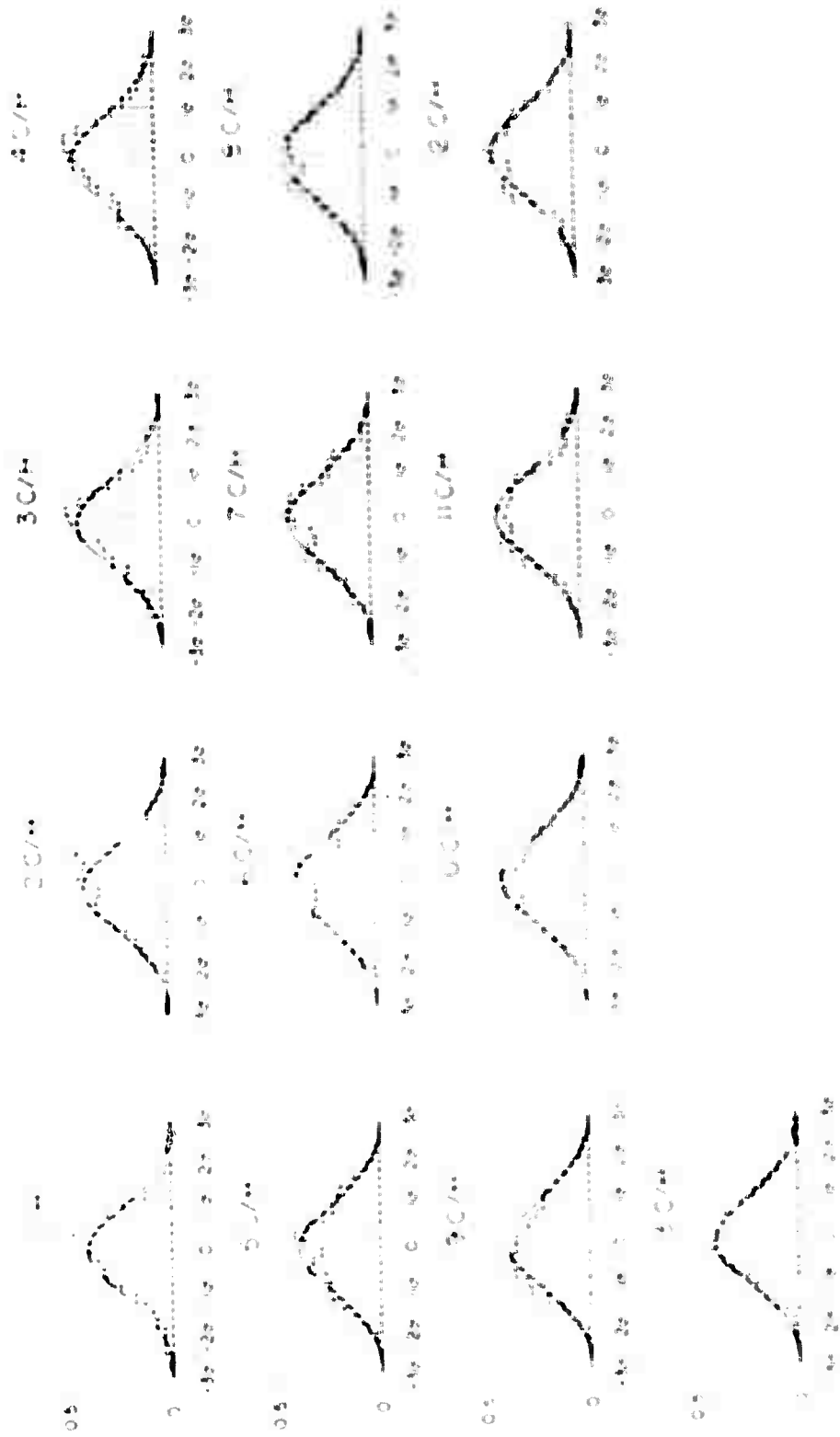


FIGURE 14. Plot of intercepts as function of frequency.



When the data was broken into a couple of pieces, some indication of nonstationarity was found. However this fact is not too significant since one would anticipate that for longer records some seasonal variation is introduced.

Conclusion

From the results one can conclude that ionospheric background motions in the frequency range covered by the analysis respond to an excitation mechanism which has a main component of 24 hours (diurnal) and a smaller one of 12 hours (semidiurnal).

The reason why the model cannot explain much of the variance in the frequency range $3 \text{ CMI} < f < 6 \text{ CMI}$ is probably that this range also corresponds to the Brunt-Väisälä frequency at ionospheric altitudes. It is well known that the Väisälä frequency besides being the cutoff frequency for internal gravity waves is also the local value of the resonant buoyancy frequency, and it should be expected that locally generated disturbances will show up strongly in this frequency range.

No particular patterns have been found in the distribution of the residuals. From analysis of smoothed histograms computed from the residuals they appear to be normally distributed.

Finally it must be pointed out that a similar analysis carried out using data from a long period microbarograph showed that no more than 9% of the variance could be explained by the model.

5. IONOSPHERIC RESPONSE TO INTERNAL GRAVITY WAVES RELEVANCE TO DOPPLER OBSERVATIONS

The response of the F region of the ionosphere to internal gravity waves is anisotropic due to the presence of the geomagnetic field. Hooke (1970) has shown that the response of the ionosphere besides being anisotropic is also subjected to diurnal and seasonal changes. This author describes the response in terms of electron density variations, which are not directly related to the doppler observations. Since the phase-path sounders are assumed to measure the vertical velocity of the electrons, it was decided to make a simplified model of the response in terms of this parameter to aid in the interpretation of our observations.

Bugnolo (1967) has shown that the vertical velocity of the electrons when no external electric fields are present can be expressed in terms of the velocity of the neutral gas by the uncoupled equation

$$V_{ze} = -\cos(a) \cos(b) \cos(\phi_g) \cdot U + \cos^2(b) \cdot W \quad (14)$$

where V_{ze} is the vertical velocity of the electrons; the parameters refer to Figure 16 and are

- a = angle between magnetic south and H_0 ,
- b = angle between the vertical axis of coordinates and H_0 ,
- ϕ_g = angle of arrival of the neutral wave measured from magnetic north,
- H_0 = the main geomagnetic field,
- U = horizontal particle velocity of the neutral gas,
- W = vertical particle velocity of the neutral gas

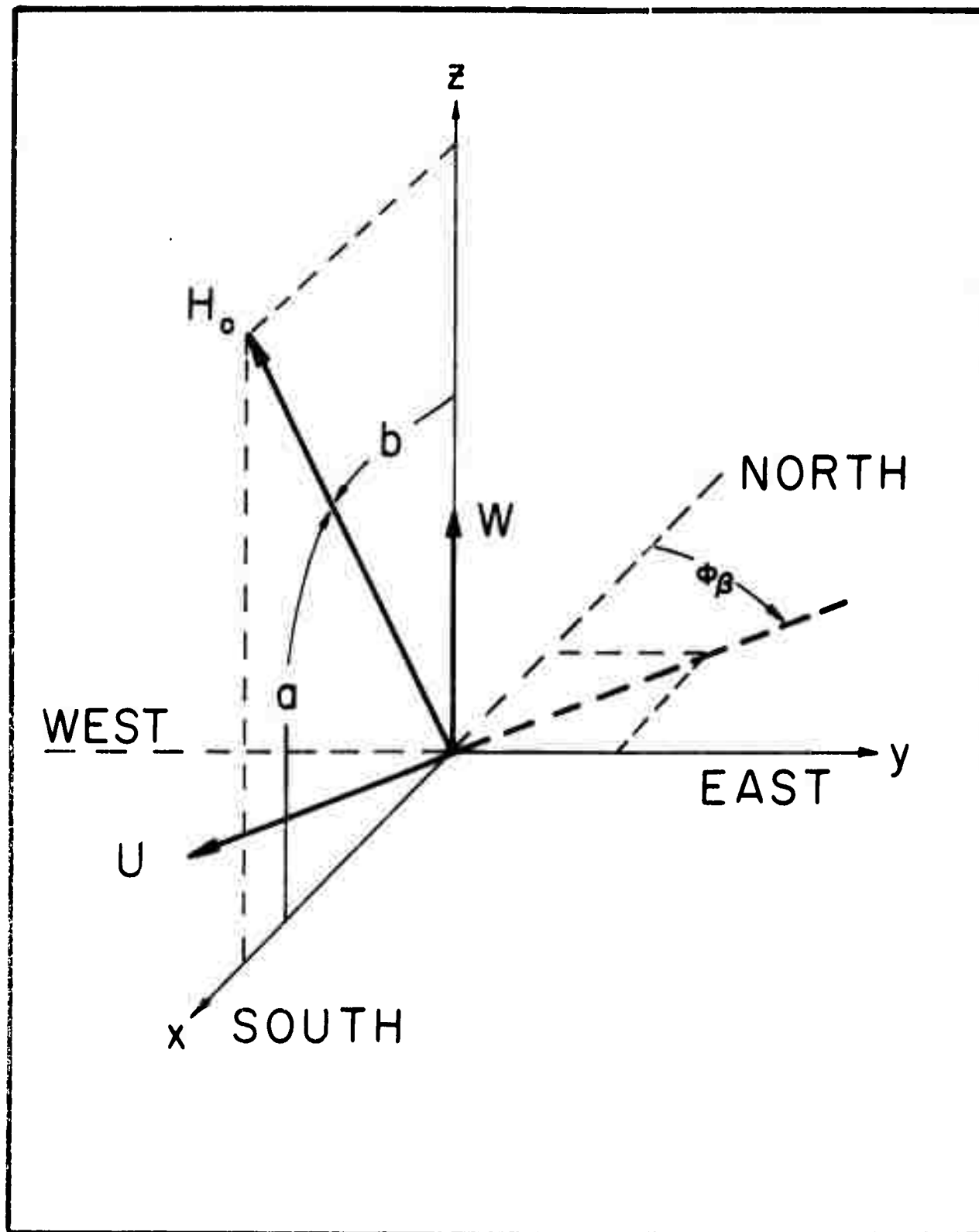


FIGURE 16. Geometry for ionospheric response.

In the northern hemisphere

$$\begin{aligned} a &= \Pi - I \\ b &= \pi/2 + I \end{aligned} \quad (15)$$

where I is the magnetic dip angle.

The values of U and W can be obtained from equations (4.2) and (4.8) of Tolstoy (1963) in the form

$$\frac{U}{W} = \frac{-i\alpha}{\omega^2/C^2 - \alpha^2} (\nu^2 - g/C^2) \pm i\gamma \quad (16)$$

where the \pm sign in front of γ indicates upgoing or downgoing waves, and

$$\begin{aligned} \alpha &= \text{horizontal wave number,} \\ \omega &= \text{angular frequency of the internal wave,} \\ C &= \text{speed of sound,} \\ \nu &= 1/2H, H \text{ being the scale height,} \end{aligned}$$

and

$$\gamma^2 = \frac{\omega^2}{C^2} - \alpha^2 + \frac{\alpha^2}{\omega^2} N^2 - \nu^2, \quad (17)$$

with

$$N = \text{Väisälä frequency}$$

Using equations (15) and (16) the response of the ionization can be written in the form

$$R = \frac{V}{W} = -\cos I \cdot \sin I \cdot \cos \phi_\beta \cdot F + \sin^2 I \quad (18)$$

Where $F = U/W$.

It should be pointed out that R is a complex number and as such it has amplitude and phase.

Taking typical values for the various parameters and noting that the magnetic dip angle at Westwood is 72° , one obtains the responses shown in Figures 17 through 21.

Inspection of the figures indicates that for horizontal wavelengths of 400 km there is a large phase shift between the electron vertical velocity and the neutral gas vertical velocity for arrivals having azimuths between -60° and 60° while the amplitudes are about a factor of three smaller for arrivals coming from the south. Smaller wavelengths are less affected by this anisotropy. The amplitude response curves compare well with those of Hooke (1970). Inclusion of the solar control on the production of ionization will distort these response curves in the same way as those shown by Hooke (1970). The curves shown here would correspond to noontime conditions.

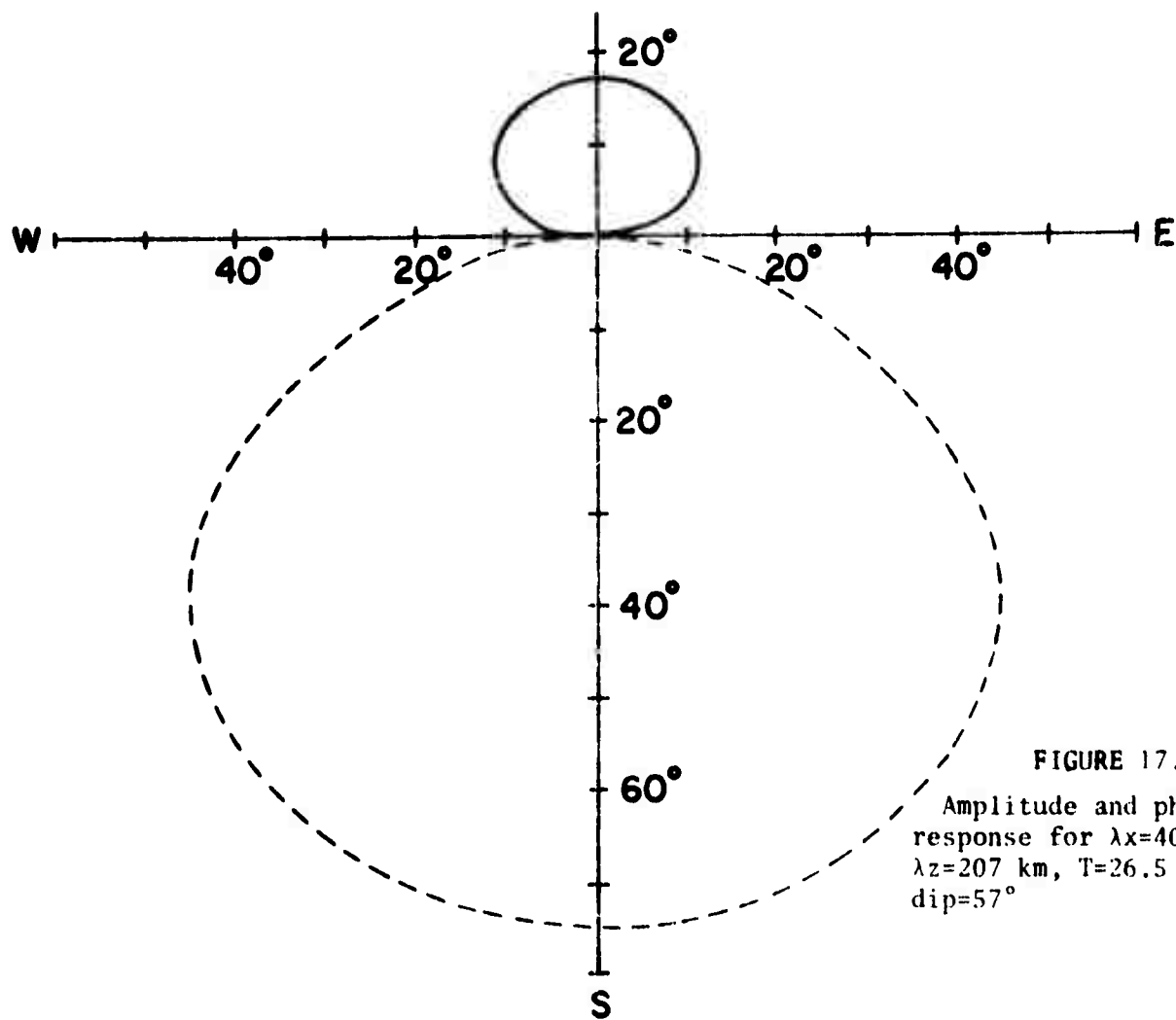
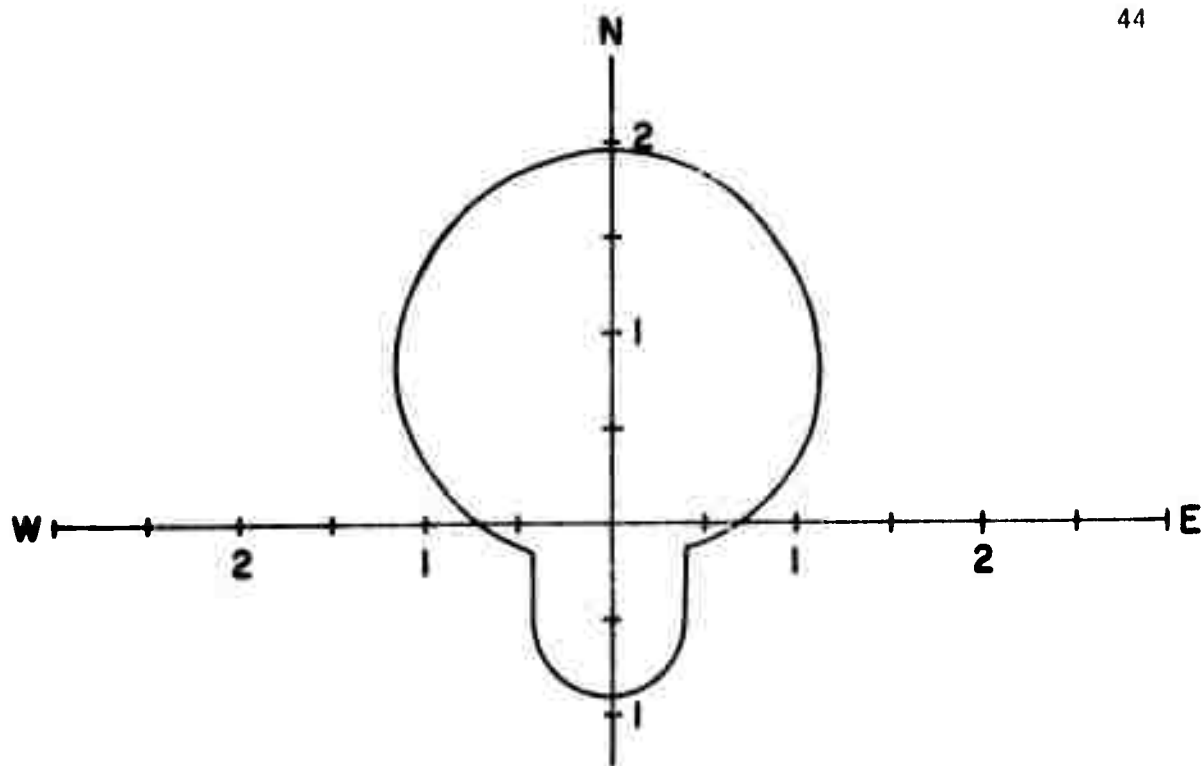


FIGURE 17.

Amplitude and phase
response for $\lambda x = 400$ km,
 $\lambda z = 207$ km, $T = 26.5$ min
dip = 57°

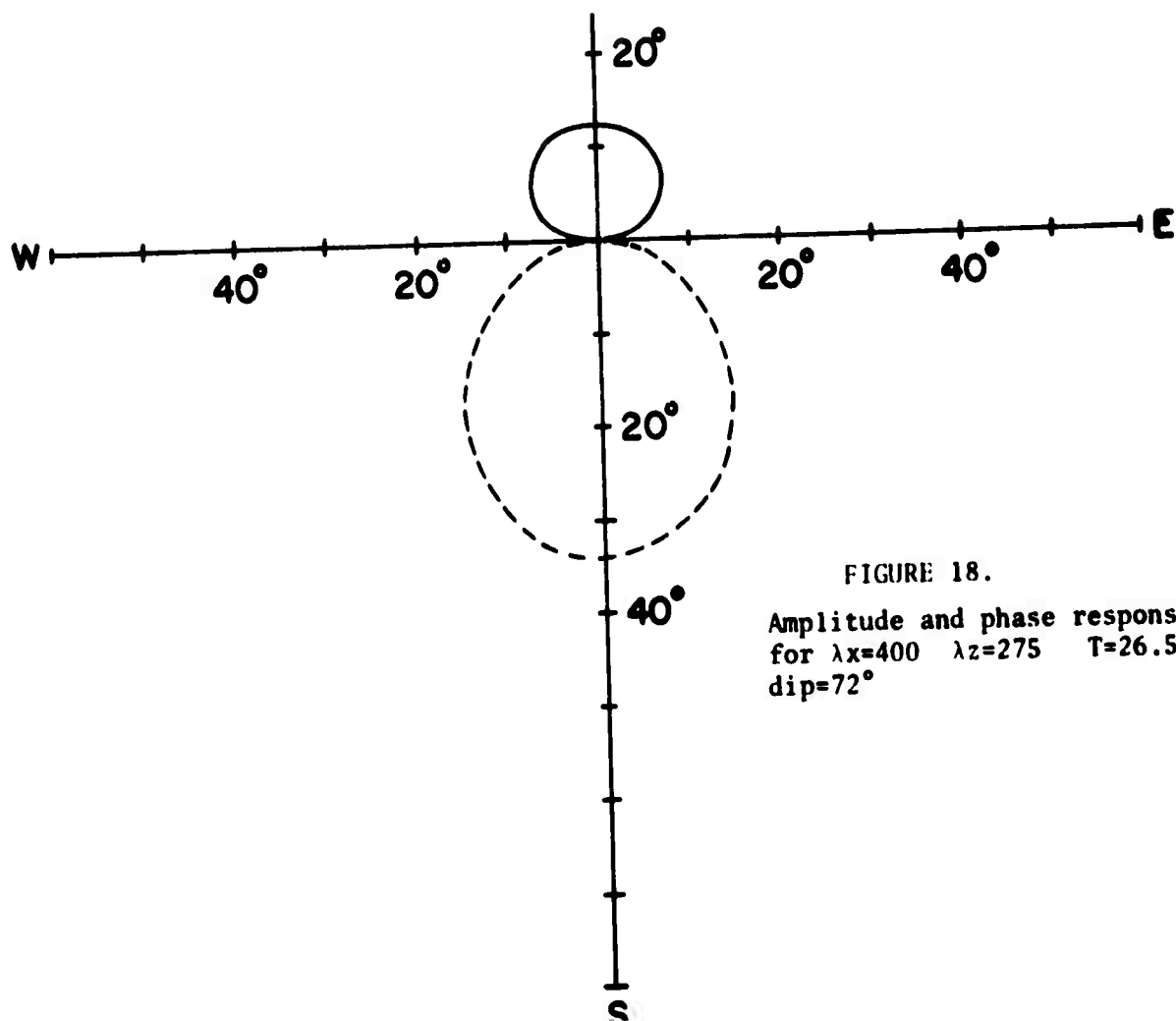
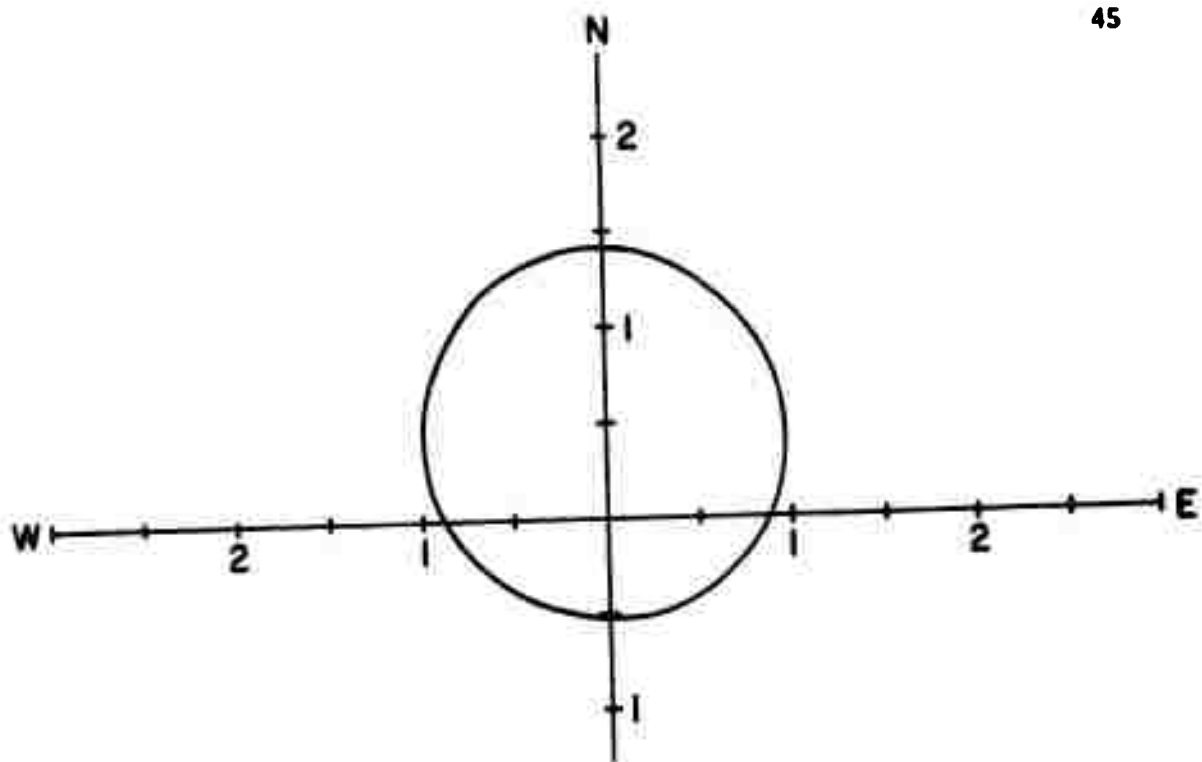


FIGURE 18.
Amplitude and phase response
for $\lambda_x=400$ $\lambda_z=275$ $T=26.5$
dip= 72°

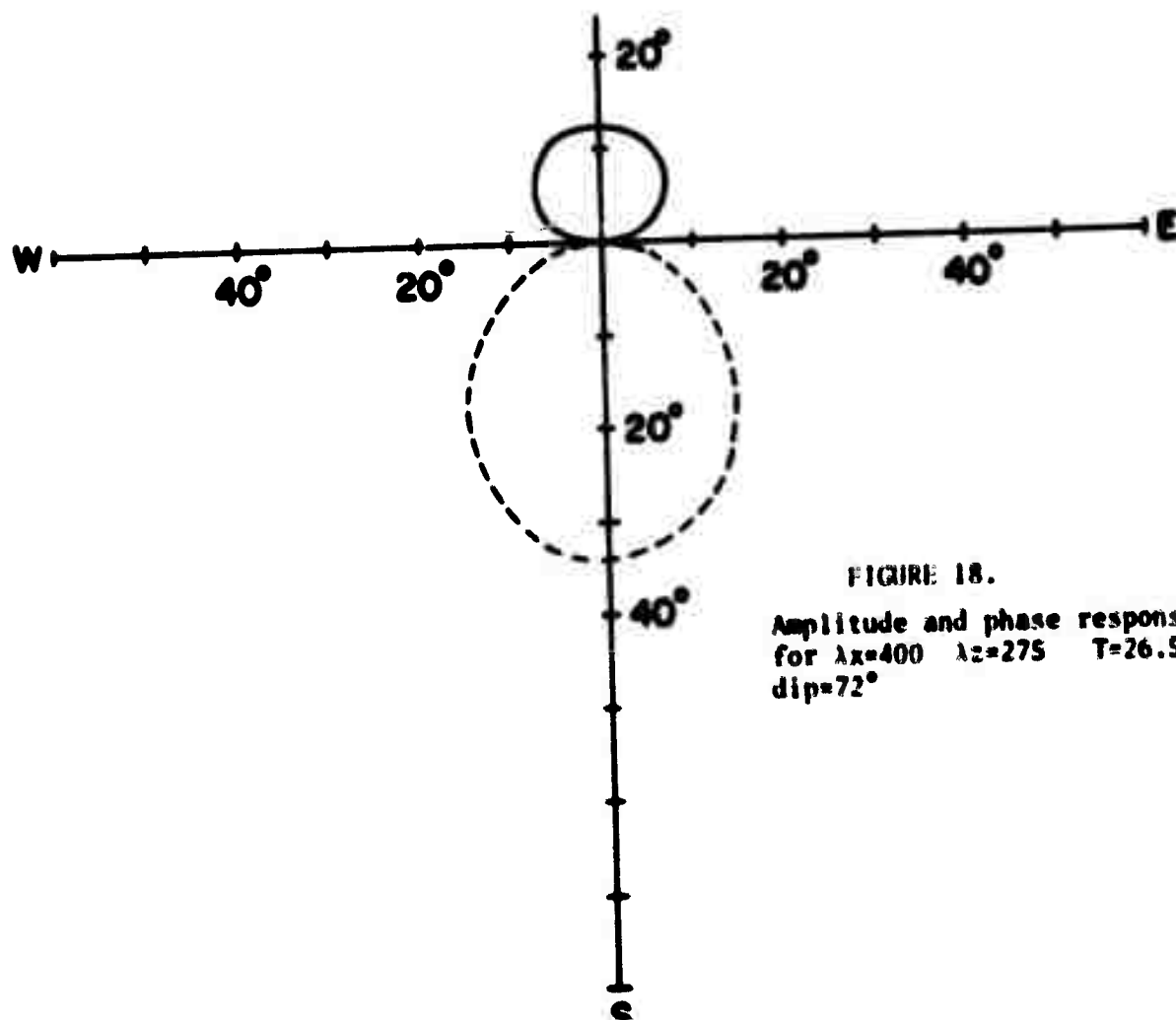
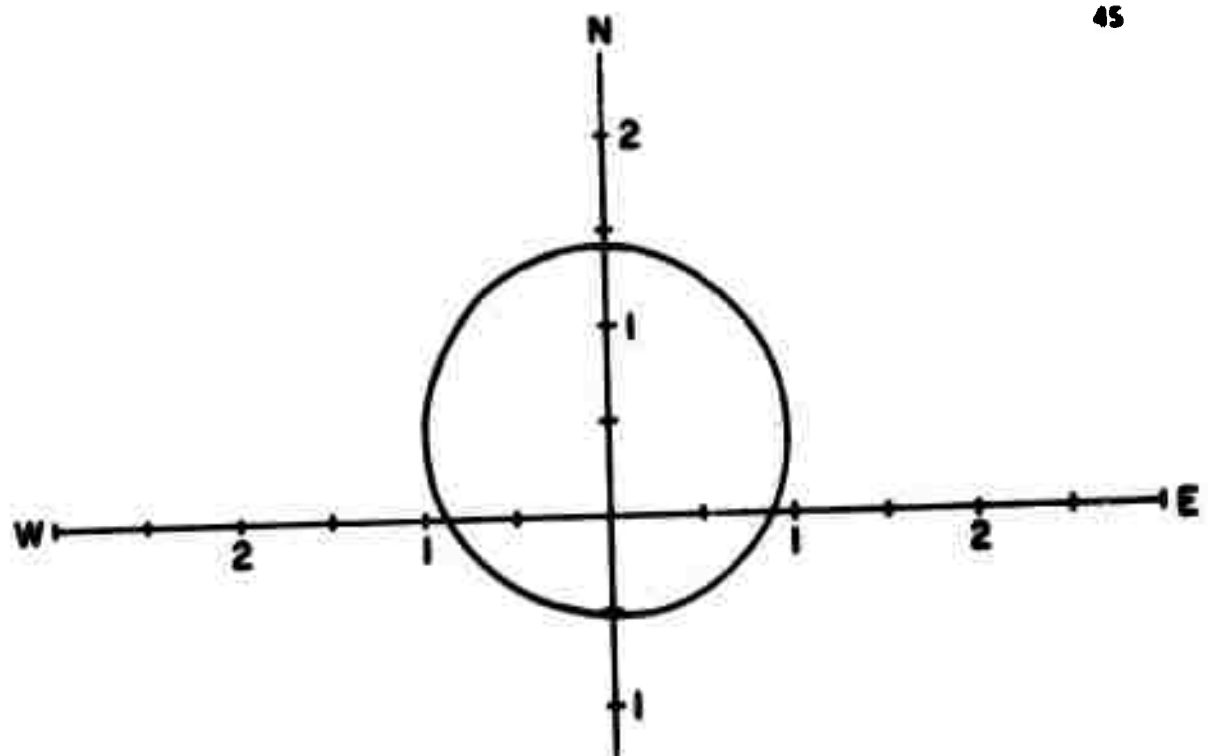


FIGURE 18.
Amplitude and phase response
for $\lambda_x=400$ $\lambda_z=275$ $T=26.5$
dip=72°

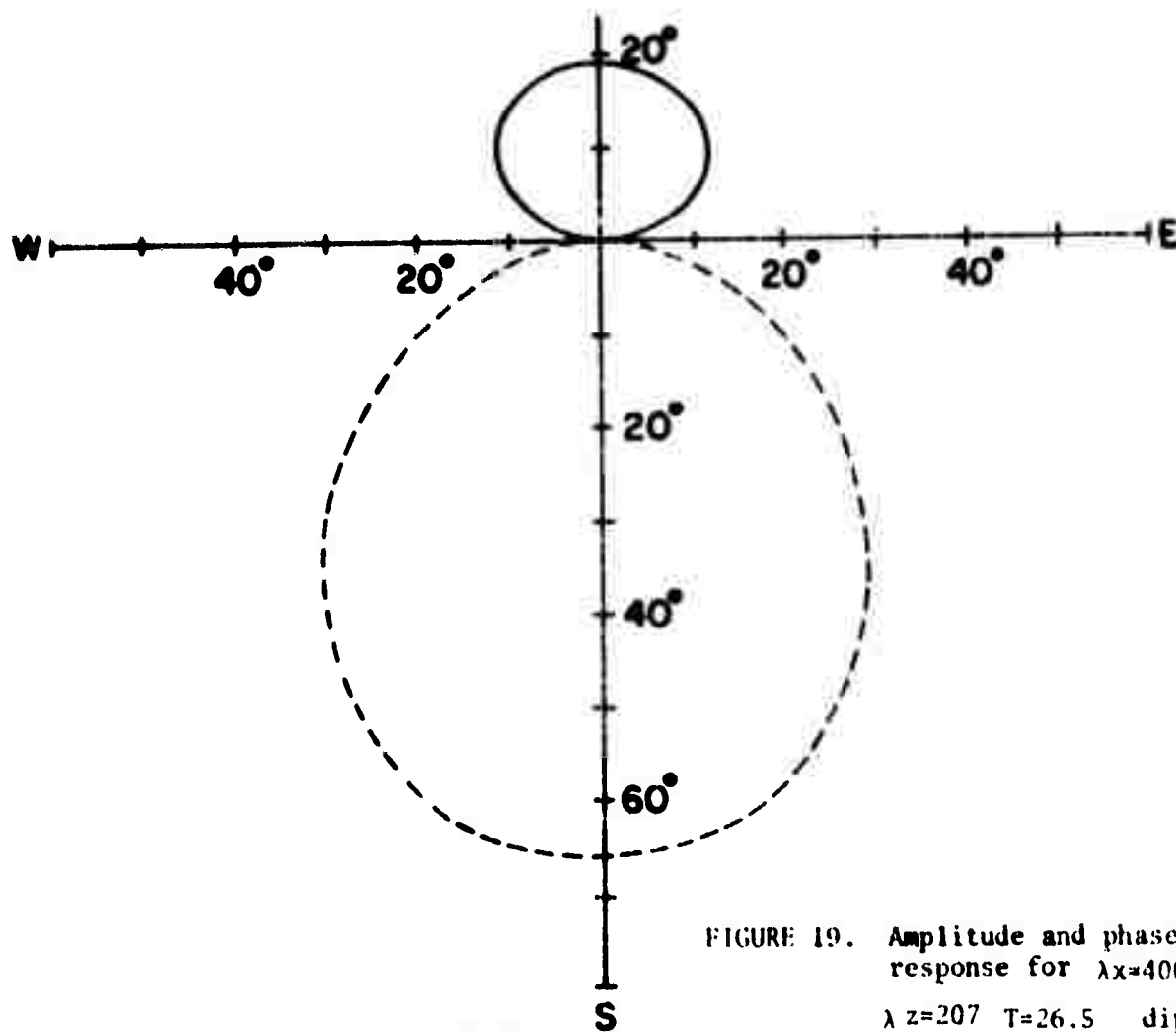
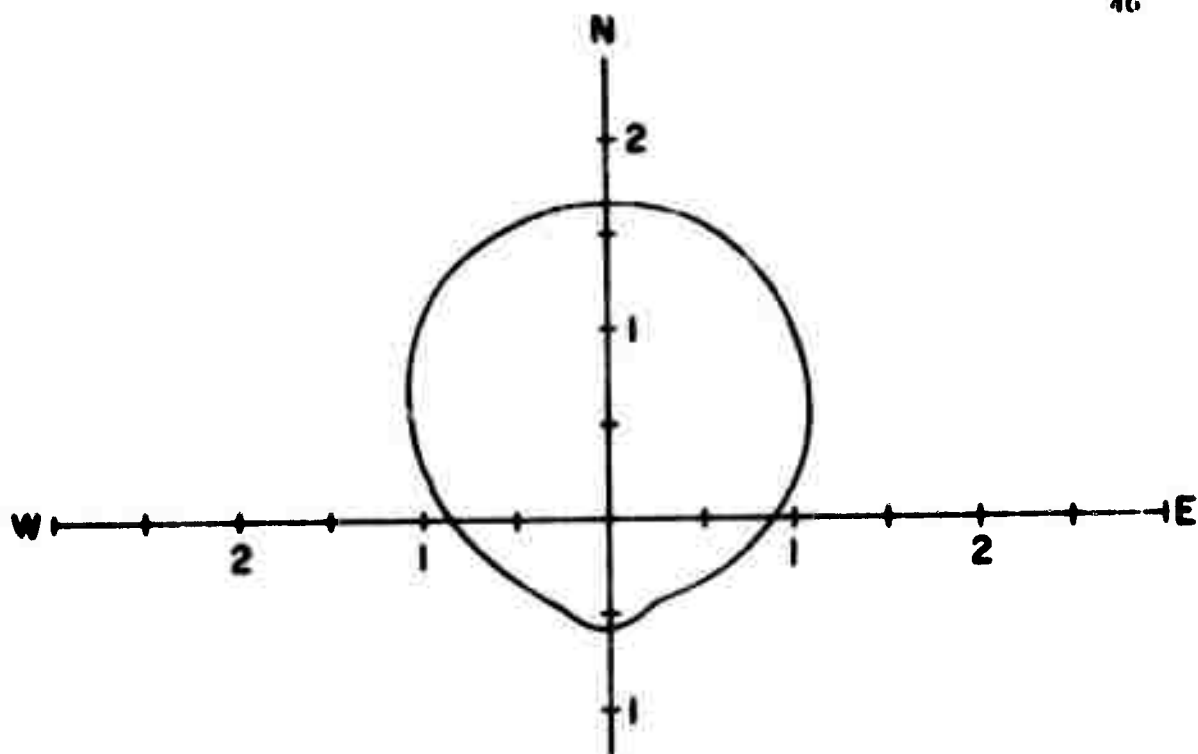


FIGURE 19. Amplitude and phase response for $\lambda x = 400$

$\lambda z = 207$ $T = 26.5$ $\text{dip} = 72^\circ$

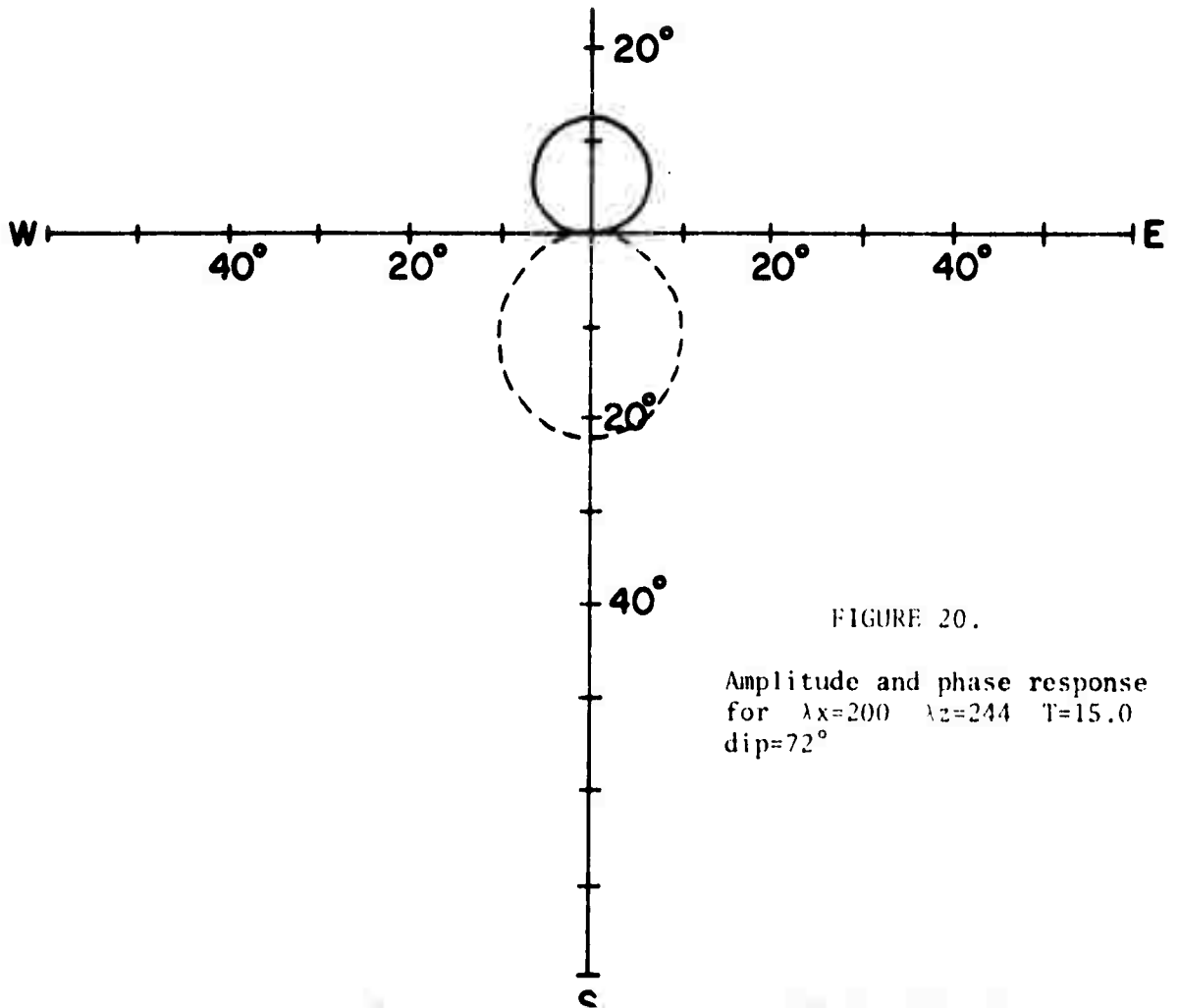
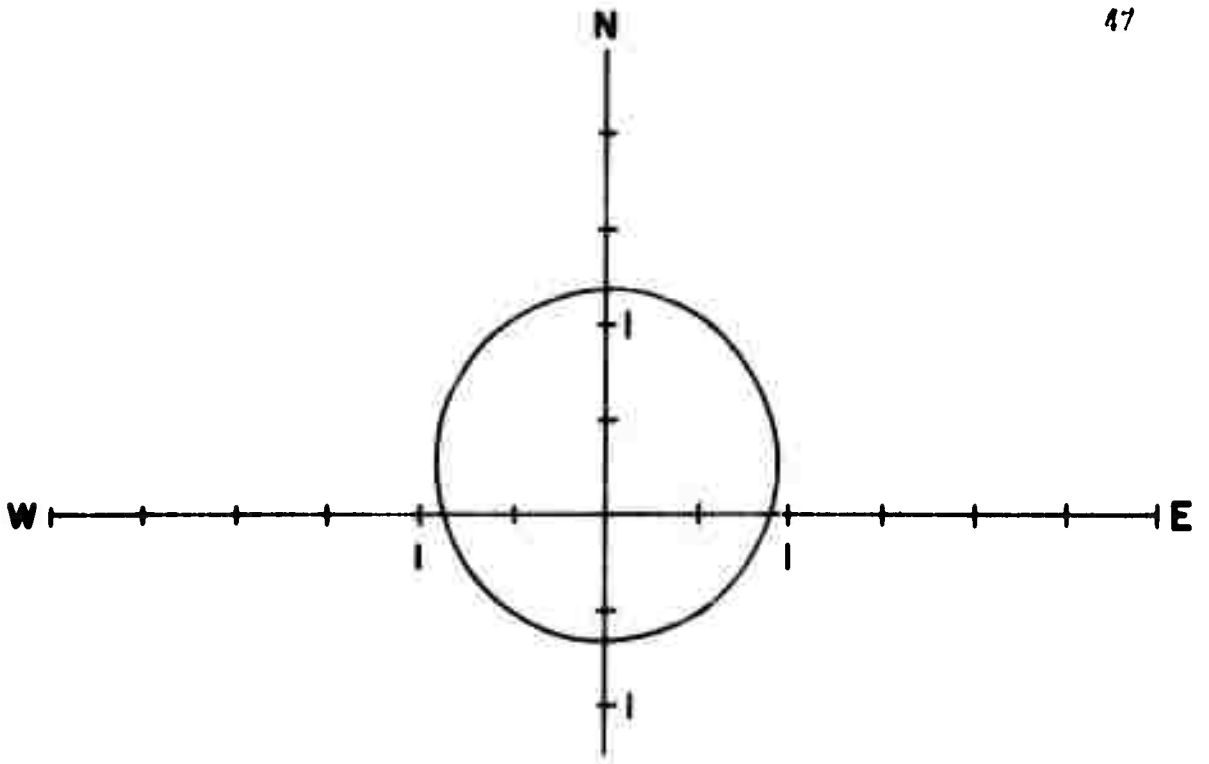


FIGURE 20.

Amplitude and phase response
for $\lambda_x=200$ $\lambda_z=244$ $T=15.0$
dip= 72°

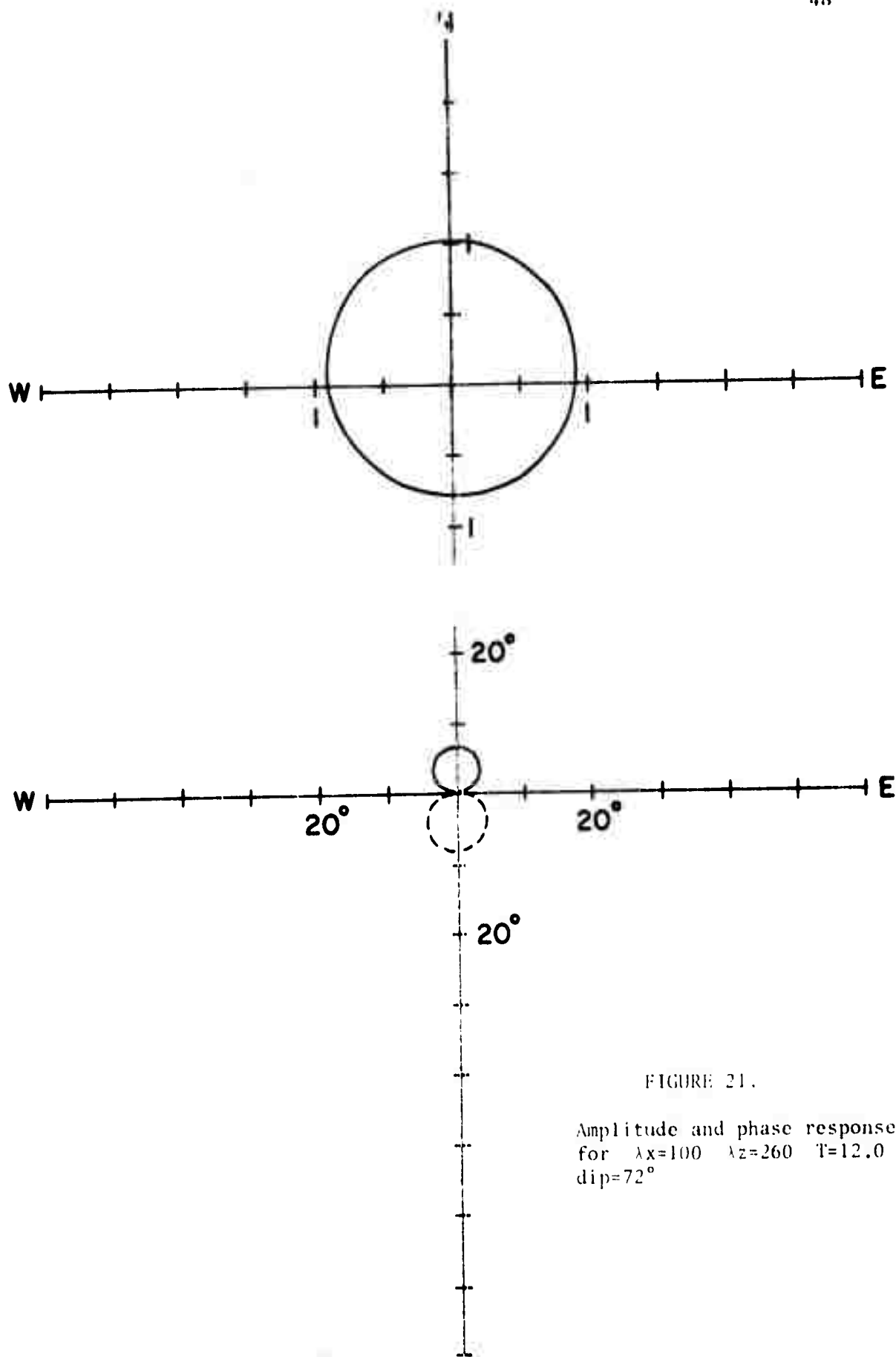


FIGURE 21.

Amplitude and phase response
 for $\lambda_x=100$ $\lambda_z=260$ $T=12.0$
 dip= 72°

6. ANALYSES OF RECORDS CORRESPONDING TO HIGH-ENERGY EVENTS

During the high energy events of the summer of 1970, phase-path fluctuations data were collected by the doppler array operated by Teledyne Isotopes. Array processing of the data was accomplished by applying co-phase analyses to the records. Cophase, a statistic characterizing signal strength as a function of velocity and direction, has been defined by Posmentier and Hermann (1971).

An 80-minute time window was moved progressively in time, in steps of 15 minutes starting 3 hours before and ending 3 hours after the time at which infrasonic signals were expected over the array. The frequency band covered by the analyses was from 0.000625 Hz to 0.0020833 Hz, corresponding to periods between 8 minutes to 26.6 minutes. The width of the frequency domain window was 0.0004166 Hz.

A sample of the doppler records corresponding to some of the events is shown in Figure 22. The arrows in the figure correspond to the arrival time estimates. Examination of this figure indicates that the phase-path fluctuations have periods between 20 minutes and 60 minutes. These periods are well within the atmospheric gravity wave period range. However, the results of the array processing showed that high values of cophase occur at phase velocities and azimuths which do not agree with those that would be expected from the location of the high energy events, i.e., $450\text{-}650 \text{ msec}^{-1}$ and $238^{\circ} \pm 15^{\circ}$ respectively. These results are not difficult to understand since the estimated arrival time for the long period atmospheric gravity waves is about 2300 UT. It has been shown in Section 4 that at the time

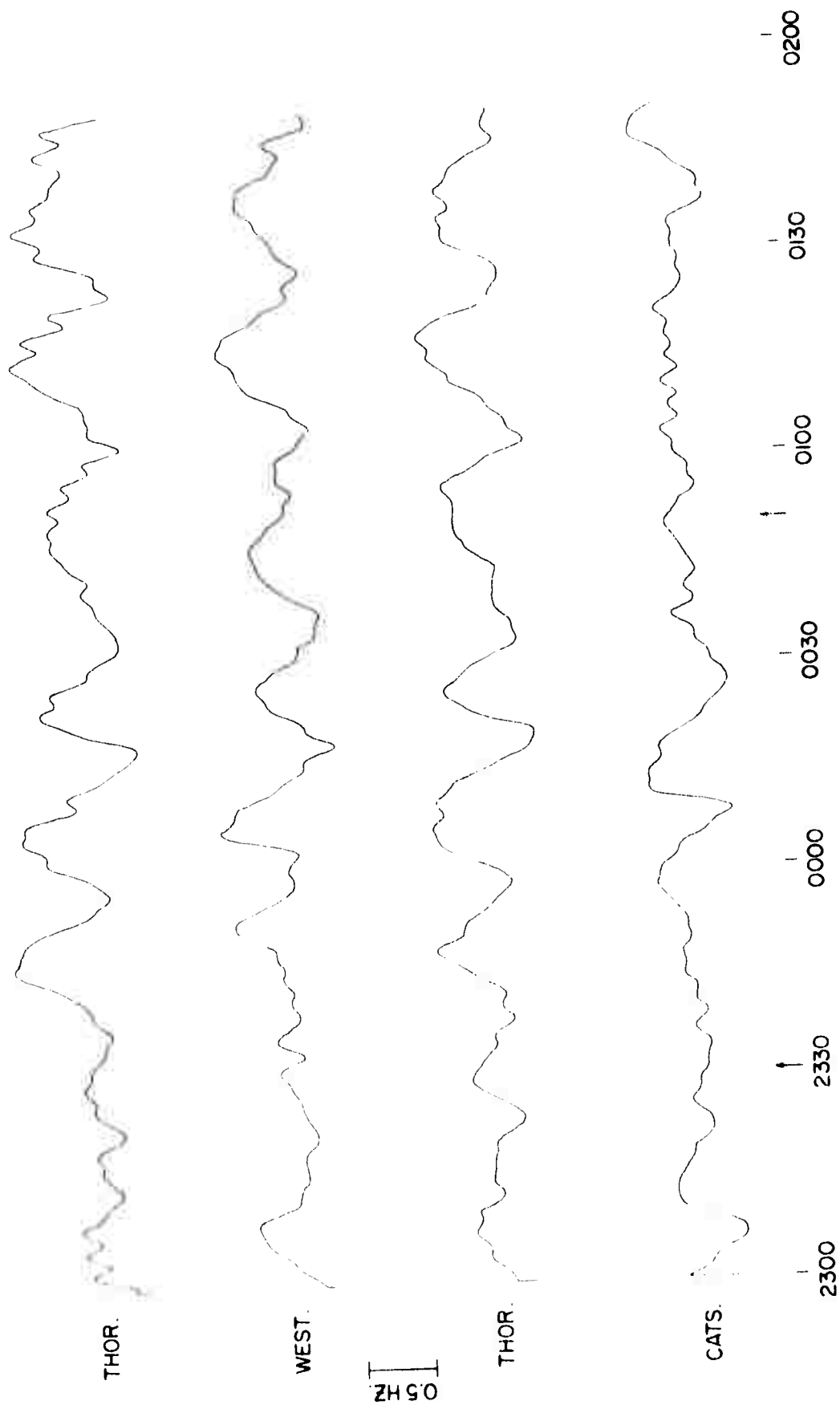


FIGURE 22. Phase-path record corresponding to high energy event of 2 August 1970.

around 2300 UT the background motion increases substantially. This is illustrated in Figure 23 where it can be seen quantitatively that the power spectra of the background motions is an order of magnitude larger at this time of day than it is during the previous hours.

This behavior of the background makes the identification of the signals coming from high energy events almost impossible. The direction of arrivals and the phase velocities obtained from the analyses suggest that the fluctuations are dominated by TID activity.

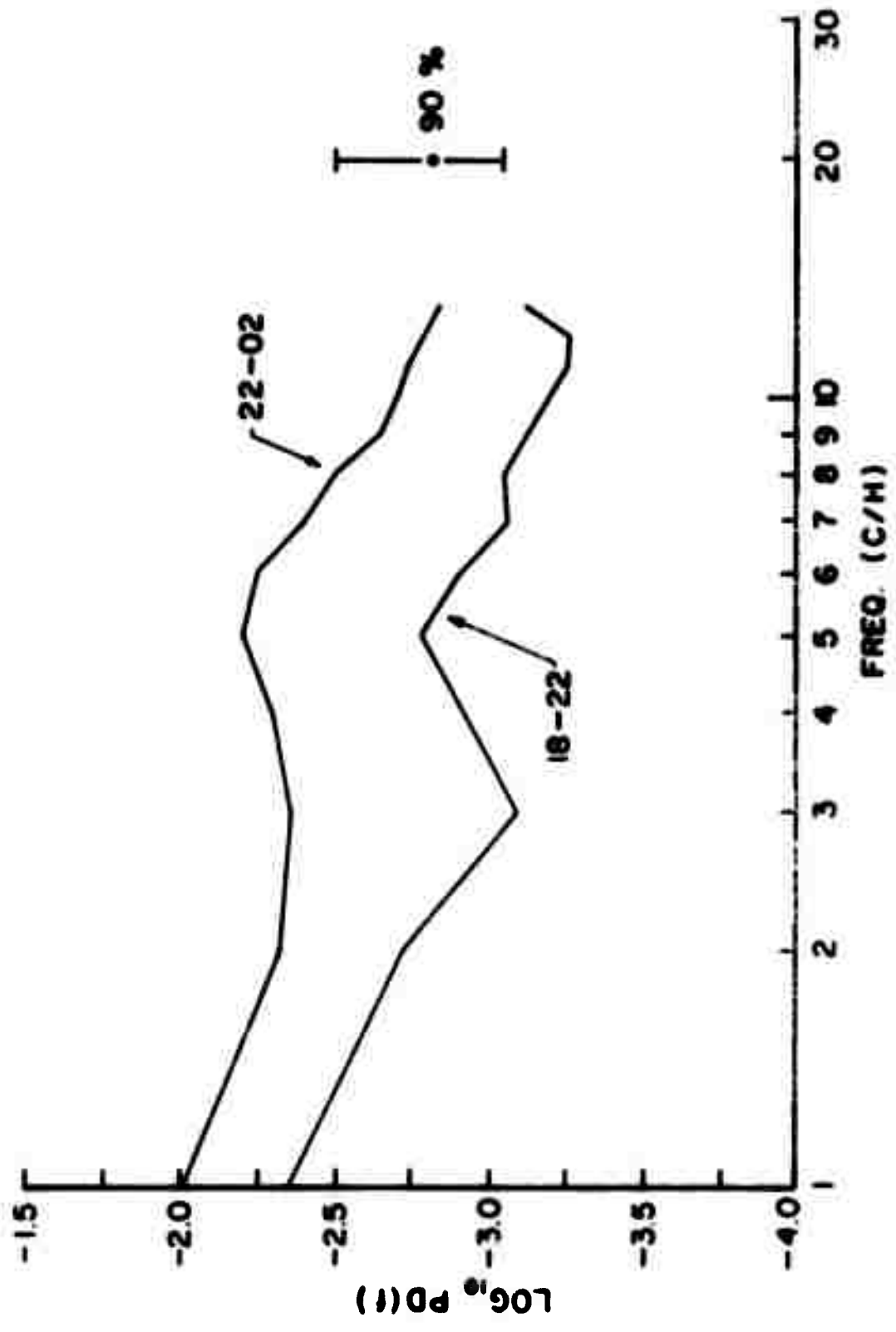


FIGURE 25. Sample power spectra showing increase of background at arrival time

7. SUMMARY

Analyses of atmospheric wave data from a long period multi-sensor array were carried out. The array was situated in the New York-New Jersey region and operated during 1969-1970. The results of the analyses enable us to draw the following conclusions.

Saturn Apollo launches generate acoustic gravity waves in the frequency range $0.00333 \text{ Hz} - 0.0111 \text{ Hz}$. These waves induce, at ionospheric heights, phase-path (doppler) fluctuations which can be detected by phase sounders at distances of the order of 1500 km from the rocket trajectory.

The direction of arrival of the signals intersects the trajectory of the rockets at a point at which the altitude of the rockets is approximately 160 km. This fact suggests that the atmospheric disturbances are generated at ionospheric levels.

Seasonal variations in the lower atmosphere do not affect these arrivals, indicating that propagation of these waves takes place in the upper atmosphere.

The phase velocities of these signals are of the order of 720 msec^{-1} while the group velocities range from 220 msec^{-1} to 450 msec^{-1} . The average horizontal wavelengths are of about 130 km.

The lack of arrivals at times corresponding to smaller rocket launchings and the wavelengths involved strongly suggest that the waves are generated by rocket engine exhaust plume rather than by a ballistic mechanism.

Analyses of ground level pressure data have failed to detect any signals corresponding to the doppler signals. This lends additional support to the hypothesis that propagation of these waves takes place at ionospheric heights.

A dispersion study of a TID was used to illustrate the potential of this type of study to provide information about the neutral structure of the upper atmosphere. The horizontal phase velocities of this TID event ranged from 120 msec^{-1} to 240 msec^{-1} . A criterion given by Tolstoy (1971) was used to test the proximity of the observation point to the turning point of the internal gravity wave, and then a WKB approximation was used to compute the various modes.

It is possible to construct parametric models to describe the time variation of the ionospheric background motions. It was found that in motions the frequency range 1 to 13 CPH respond to an excitation mechanism having periodicities of 12 and 24 hours. Between 36% and 50% of the variance can be explained by the model.

A simple model for computing ionospheric response to internal gravity waves showed that for horizontal wavelengths of the order of 400 km, at a magnetic dip angle of 72° , the amplitudes of the doppler variations can vary by a factor of 3 depending on the direction of the wave. For smaller wavelengths this anisotropy is less.

Finally, the analyses of data corresponding to high energy events showed that at the expected arrival time (2300 UT) the doppler background level increases by an order of magnitude, making the identification of the signals almost impossible.

8. PAPERS PUBLISHED AND IN PREPARATION

The following papers have been published during this year:

Cophase: An ad hoc array processor.

E. Posmentier and R. Herrmann

Journal of Geophysical Research, 76, 2194, 1971

Phase-height fluctuations in the ionosphere between 130 and 250 km.

I. Tolstoy and H. Montes

J. Atmosph. Terr. Phys. 33, 775, 1971

Cophase analyses of atmospheric wave data.

H. Montes and E. Posmentier

Geophysical Journal of the Royal Astronomical Society,
26, 1972.

The following papers are in preparation:

Dispersion of TIDs.

H. Montes and I. Tolstoy

Parametric model of ionospheric background motions.

H. Montes and M. Hinich

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